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A framework for modeling spatial node density in waypoint-based mobility

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Abstract User mobility is of critical importance when designing mobile networks. In particular, "waypoint" mobility has been widely used as a simple way to describe how humans move. This paper introduces the first modeling framework to model *waypoint-based mobility*. The proposed framework is simple, yet general enough to model any waypoint-based mobility regimes. It employs first order ordinary differential equations to model the spatial density of participating nodes as a function of (1) the probability of moving between two locations within the geographic region under consideration, and (2) the rate at which nodes leave their current location. We validate our model against real user mobility recorded in GPS traces collected in three different scenarios. Moreover, we show that our modeling framework can be used to analyze the steady-state behavior of spatial node density resulting from a number of synthetic waypoint-based mobility regimes, including the widely used Random Waypoint model. Another contribution of the proposed framework is to show that using the well-known *preferential attachment* principle to model human mobility exhibits behavior similar to random mobility, where the original spatial node density distribution is not preserved. Finally, as an example application of our framework, we discuss using it to generate steady-state node density distributions to prime mobile network simulations.

Keywords Realistic mobility models · Ordinary differential equations · Modelling · Spatial node density

Part of this work was done while Bruno was at UCSC

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1 Introduction

When designing and evaluating wireless networks and their protocols, user mobility is a critical consideration. So much so that user mobility has inspired an extensive body of work both in infrastructure-based networks (e.g., wireless LANs or WLANs), as well as in infrastructure-less networks, a.k.a., wireless, self-organizing networks (WSONs). The latter include wireless mobile ad-hoc networks (MANETs), wireless sensor networks (WSNs), and disruption-tolerant networks (DTNs). Unlike their infrastructure-based counterparts where only end user nodes are mobile, in infrastructure-less networks, every node may move and thus mobility plays a considerable role in the performance of the network.

Synthetic mobility regimes are an important consideration on simulating, testing and conducting performance evaluation on wireless networks and their protocols. The research community have been investing quite a lot of effort in developing mobility models that would reflect more faithfully the mobility patterns and characteristics found in real mobile applications. When moving in real mobility scenarios (e.g. walking on a park, city center, university campus, etc), humans do not behave randomly, but tend to form groups and clusters, even when moving independently of each other. These clusters are formed due to the social interactions between the mobile entities, geographical restrictions in the area, the intrinsic attraction some specific locations might have towards some nodes, etc. One way to characterize and describe mobility, and study how mobile entities interact and agglomerate is through the *spatial density* of mobile nodes. Spatial node density can be defined as the number of nodes located in a given unit area and has significant impact on fundamental network properties, such as connectivity and capacity, as well as on core network functions, e.g., medium access and routing.

Yet, the characterization of real human mobility through spatial node density remains a challenging subject. To date, only a few efforts have focused on modeling spatial density. Notable examples include [15, 5, 26]. However, most previous work have been focusing exclusively on synthetic mobility regimes, specially the Random Waypoint (RWP) model [6], since it is the most used mobility model in the literature, due to its simplicity and easy of implementation.

In this paper, we focus on modeling the spatial node density of "waypoint"-based mobility. More specifically, our model describes the spatial density steady-state behavior under waypoint-based mobility which is a mobility pattern characterized by having nodes probabilistically choose the next destination, or waypoint, based on some probability density function, moving to this point with a given speed, pausing for some time, and starting the process again. We define spatial density as the percentage of subareas (or cells) containing $\geq k$ nodes, which can also be viewed as the probability of finding a cell with k or more nodes at a given time. We assume a Markovian property for this quantity as the count of the number of users in each cell (our state) at the next time instant only depends on the number of users in the cells in the current time instant, given the cost of making a transition (increasing or decreasing the number of users in each cell), and the rate at which transitions occur. We present an approximation by a set of Ordinary Differential Equations (ODEs) and propose a framework to mathematically model spatial node density under different "waypoint"-based mobility regimes.

We contend that waypoint-based mobility is one way to describe forms of human mobility. Therefore, we apply our model to describe the steady state of real human mobility and validate it against real user mobility recorded by GPS traces in different scenarios, comparing the results against the corresponding traces. Moreover, we present comparative results for steady-state spatial distribution analysis of a number of synthetic waypoint mobility regimes. To the best of our knowledge, this is the first node density modeling framework generic enough that it can be applied to any waypoint-based mobility regime. As an example, we use our framework to model the well-known RWP mobility regime. Our model confirms the well-known result showing

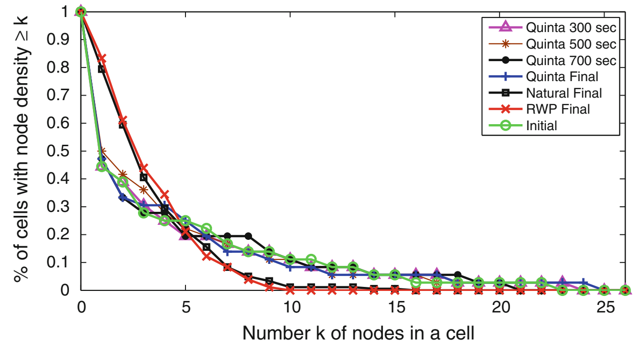

Fig. 1 Node spatial density distribution at different trace collection times for mobility in a city park

that node density's steady-state behavior under RWP mobility tends to homogeneity, as defined in [7][1].

Furthermore, several previous work on synthetic mobility modeling (discussed in more detail in the next section) apply the *preferential attachment* principle [1], in order to create and maintain the formation of clusters of mobile nodes. We also use our framework to model waypoint-based mobility regimes that apply the preferential attachment principle. We show through the application of our proposed model that using preferential attachment to model human mobility leads to undesirable steady-state behavior. More specifically, our model shows that, at steady state, the original spatial node density distribution is not preserved and exhibits behavior similar to random mobility a la Random Waypoint regime. This behavior has been observed empirically in [27]. Instead, real human mobility exhibits "persistent" density heterogeneity as illustrated in Fig. 1. This figure shows the spatial density distribution for one of the traces used in this paper which was collected in the Quinta da Boa Vista Park in Rio de Janeiro, Brazil.

The first 4 curves in the plot refer to the distributions at instants 300, 500, 700, and final (900 s), which is the end of the trace collection interval. The two other curves correspond to the node density distributions measured after 900 s of simulations of two synthetic waypoint-like mobility regimes, namely RWP and Natural [8]. Each of these curves reflect the final node distribution averaged over 10 runs of simulations[2]. Both mobility models and the experiments that generated these curves are discussed in detail in Sect. 5. The last curve shown in the graph is the initial node distribution

[1] The use of the term "homogeneous node distribution" refers here to the fact that there is no significant concentration of nodes (clusters), and should not be mistaken with uniform distribution normally used to model the choice of next destination, speed and pause time in random mobility models.

[2] A 90 % confidence level was computed. The confidence interval was too small to be seen in this scale and was omitted for clarity of the plot.

measured from the Quinta trace and also used as the initial distribution in all simulation runs of both synthetic mobility regimes studied.

From Fig. 1, we observe that the density distribution of the real trace does not vary much with time: the largest deviation from the initial distribution for any value of k at any instant is 8.3 %; the average deviation from the initial distribution measured in all the instants for all values of k is 1.27 %. Similar observations can be drawn from the other traces used in our work as reported in Sect. 4.1. Moreover, we also observe a very different behavior when applying either one of the synthetic regimes to the same scenario. They deviate significantly from the initial conditions.

However, an interesting observation here is the fact that using a preferential attachment based regime such as Natural, does not preserve the original clustering of the nodes. In fact, spatial density resulting from preferential-attachment based waypoint mobility "deteriorates", at steady state, to behavior similar to random mobility. In Sect. 5, we present more details on these results and apply our proposed model to study the steady-state behavior of spatial density of these mobility regimes.

Overall, the contributions of our work are many-fold: (1) we introduce the first spatial node density modeling framework for waypoint mobility regimes, (2) we apply our proposed framework to study the steady state behavior of real human mobility in three different real mobility scenarios, (3) we present results from applying the proposed framework over two different waypoint-based mobility models, (4) we use our model to show that the steady-state behavior of node density under preferential-attachment based mobility does not preserve node density's original distribution and exhibits behavior similar to random mobility, and (5) as an example application of our framework, we discuss using it to generate steady-state node density distributions to prime mobile network simulations.

The remainder of this paper is organized as follows. Section 2 places our work in perspective by presenting related work in mobility modeling and characterization. Our ODE model is presented in detail in Sect. 3, how its parameters are set, and our implementation. Section 4 show the validation of our proposed framework towards modeling real human mobility, while Sect. 5 presents the applications of our work on modeling spatial density of synthetic waypoint-based mobility regimes. Finally, Sect. 6 concludes the paper with a discussion of future work.

2 Related work

Mobility models are vital to the design, testing, and evaluation of wireless networks and their protocols. As an indication of the importance of mobility models to the study of wireless network protocols, most well-known network simulators include "mobility generators", which, following a pre-specified mobility regime, determine the position of network nodes over time during simulation runs. Synthetic mobility generators have been extensively used in the study of wireless networks [9]. A notable example of such synthetic mobility models is the Random-Waypoint Mobility (RWP) regime [6].

The work by Bettstetter et al. [7] points out that random mobility leads to homogeneous node distributions. They proposed a method that creates initial non-homogeneous node distributions and in [15], analyze via simulations the impact of random mobility in maintaining the non-homogeneity of spatial node density distributions. They also propose a metric for measuring such non-homogeneity as well as a variant of RWP mobility that maintains the non-homogeneity of an original node distribution.

More recently, network researchers and practitioners have been trying to use more realistic scenarios to drive the evaluation of wireless network protocols. This motivated initiatives such as the CRAWDAD [11] trace repository, which makes real traces available to the networking community. These traces can then be used to run trace-driven simulations. Even though initiatives like CRAWDAD have greatly increased availability of real traces, relying exclusively on traces to design and evaluate network protocols would not allow a broad enough exploration of the design space.

To address this problem, a number of efforts have proposed mobility models based on realistic mobility patterns [25]. Notable examples include [3, 4, 31, 17]. More recent work focuses on the "scale-free" properties observed in many real networks like the Internet, the Web, and some social networks, to name a few. The seminal work of Barabási and Albert [1] proposes a model that generates scale-free networks, i.e., networks whose node degrees follow a power law distribution. They demonstrate that many real-world networks are scale free, that is, the node degree in the network graph follows a power law and discuss the mechanism responsible for the emergence of scale-free networks. They argue that understanding this problem will require a shift from modeling network topology to modeling "network assembly and evolution". To this end, they define the Barabási-Albert model based on *growth* and *preferential attachment*. Growth refers to the fact that the number of nodes in the network increases over time, where a new node is placed with m edges connecting it to other m nodes. Preferential attachment means that a node will choose to connect to another node i with probability $\Pi(k_i) = \frac{k_i}{\sum_j k_j}$ based on the degree k_i of node i and any node j connected to node i. In other words, the *preferential attachment* principle states that “the more connected a node is, the more likely it is to receive new links”. Several recently proposed mobility models (e.g., [18, 20, 8, 23, 24, 21]), try to mimic real human mobility by following the preferential attachment principle: they define *attraction points*, whose probabilities of attracting other nodes increase as more nodes congregate around them. The main goal of these preferential attachment based approaches is to try to maintain the non-homogeneous characteristics of spatial node density observed in real mobility traces. This calls for models that are able to create and maintain the non-homogeneous node distributions and clustering observed in real human mobility. For example, in [8], a model based on preferential attachment has been proposed, where the choice of going towards an attraction region is weighted proportionally to the region’s popularity (i.e., the number of other nodes that chose it) and inversely proportional to the distance to it. We call this model *Natural* and use it as one of our case of studies. More specifically, we apply our framework to model Natural’s spatial node density stationary regime and show that it exhibits similar characteristics when compared to random mobility patterns such as Random Waypoint.

The work proposed in [21] is another example of a model that follows Barabási-Albert’s growth and preferential attachment principles. The authors even show a figure where they present their initial (after growth) and steady-state spatial distribution. It is possible to see how clusters dissipate and fade away over time. The same concept is also used in [23] where nodes are also driven by pre-defined social interactions. The proposed approach is validated by showing the power-law exponential decay of inter-contact times among node communities, comparing it with measurements in real traces.

One distinguishing feature of our work is the generality of our modeling framework which can be applied to any waypoint mobility regime. Waypoint mobility follows the following basic steps: (1) a node chooses its next destination following some given probability distribution; (2) moves to that destination in a straight line and constant speed; (3) pauses for some time (also following some pre-specified rule); and (4) repeats the process. Most previous work on modeling node spatial density have focused specifically on the RWP model. In [5], for example, analytical expressions are derived for the spatial density distribution that results from using the RWP model in simulations. The one-dimensional case is analyzed and an approximation for the two-dimensional case is also given. They also analyze the concept of attraction areas in a modified version of the RWP regime. One other effort that focused on modeling steady-state behavior of the RWP is described in [26]. In that work, stationary analytical expressions for node density and node speed are derived.

Our approach was inspired by classical epidemiological models [12] which allowed us to derive a framework that is not only general but also simple when compared to analytically solving Markov chains. This is because our framework is derived directly from a transaction-by-transaction Markov process modeling. We follow the analogy with epidemiology where mobile users “infect” subareas (or cells) as they move into them, and cells are “cured” as nodes move away from them, towards other destinations (susceptible to infection).

Another distinguishing feature of our approach is that it is based on Ordinary Differential Equations (ODEs). ODEs have been used to model a wide variety of networking functions and services. For example, ODEs were applied in a similar fashion to model epidemic forwarding [34] in a DTN environment. Similarly, in [13], an ODE model to analyze the performance of self-limiting epidemic forwarding mechanisms has been proposed. Similar ODE approaches have been applied to model worm propagation on the Internet [30, 35, 10], and bitTorrent file sharing [19].

Moreover, in [14] Partial Differential Equations (PDEs) have been used to model spatial node density of the RWP and Random Direction mobility regimes. An analysis of the transient behavior of the spatial node density under these two mobility regimes is described. While this work is another example of efforts that focus on studying random mobility, our approach is generic enough that can be used to study any waypoint-based mobility regime (including random approaches and preferential-attachment based regimes). The work described in [16] proposes a Markovian based mobility model with the purpose of forming and dissolving clusters of nodes. They study analytically spatial distribution of nodes, presenting results, specifically for their mobility model.

3 Proposed model and framework

Our objective is to model the spatial node density of a mobile network. We assume a waypoint-based mobility pattern, where nodes stay in a given location i for a given period of time and choose to leave i towards another location j with probability p_{ij}. Once the node arrives at j, the process restarts.

3.1 ODE framework

Assume a mobile network composed of m mobile nodes, where all nodes are capable of moving around inside a delimited area a. Now assume this area is divided into equally sized square subareas of size $l \times l$, defined here as *cells*. The mobile nodes can then choose to move from cell to cell with a given probability. Let $X(t)$ be the stochastic process that determines which cell a mobile node chooses at time t. We can write then $p_{ij} = P\{X(t) = i \mid X(t+\gamma) = j\}$, as the *transition probability*, which is the probability that a node in cell i, at time t, is going to choose to go to cell j at time $(t+\gamma)$, after some time step γ.

Thus, we are interested in the average number of nodes in each cell i, represented by the component $N_i(t) \forall i \in \{1, \ldots, n\}$ of the state vector $N(t) \in \mathbb{R}^{n \times 1}$, where n is the total number of cells for the desired scenario.

The variation in the number of nodes at each cell $\dot{N}_i(t) = \frac{dN_i(t)}{dt}$ is simply the difference between nodes arriving in cell i and the ones departing from the same cell at time t, as expressed in Eq. 1.

$$\dot{N}_i(t) = \underbrace{\lambda_0 + \sum_j p_{ji}\mu_j N_j(t)}_{\text{Arriving at cell i}} - \underbrace{\left(\sum_j p_{ij}\mu_i N_i(t) + \mu_0 N_i(t)\right)}_{\text{Departing from cell i}}, \tag{1}$$

where λ_0 is the rate at which new nodes arrive in cell i from outside the system and μ_0 the rate at which nodes decide to get disconnected and leave the system, given that they are at cell i. Also, μ_i is the rate at which nodes decide to leave cell i towards another cell, which allow us to write $\mu_{ij} = p_{ij}\ \mu_i$ as the rate at which nodes in cell i decide to leave this cell towards cell j. We can also define the arrival rate in cell i as the sum of the departing rates of all nodes going from cell j to cell i, over all possible values of j, including $j = i$, since we allow transitions from a cell to another position in itself. The arrival rate is given by Eq. 2.

$$\lambda_i = \sum_j p_{ji}\mu_j. \tag{2}$$

3.2 Parameters choice, discussion and simplifications

In reality we observe that nodes prefer some cells over others and some transitions over others. The probability of choosing a destination and the rate at which nodes depart from that destination depends on how popular that destination is and what are the nodes' interests in each destination. For example, nodes moving around on a campus environment may go very often from the cafeteria to the classroom, but not so often from the cafeteria to the library. This means that $p_{\text{cafeteria,classroom}} > p_{\text{cafeteria, library}}$. Moreover, since people might tend to stay inside the library for longer than in the cafeteria, the relationship between the departure rate from this two locations might be such as $\mu_{\text{cafeteria}} > \mu_{\text{library}}$.

In order to simplify our model, more specifically the choice of the parameters (departure rates and transition probabilities), we define the rate μ_i as the inverse of the average time spent by the nodes in cell i. We also considered the transition probabilities independent of where the transition originated. This means that the probability of going from cell j to cell i is the same probability of simply choosing cell i as the next destination for all j. We then make $p_{ji} = P\{X(t) = j \mid X(t+\gamma) = i\} = P\{X(t+\gamma) = i\} = p_i$.

Moreover, in order to validate our model we have chosen to extract the model parameters from—and compare our results with—real live GPS traces, where the number of nodes in the system remains constant during the whole duration of the trace. For that reason, in the results we present in Sect. 4.3 we used a slightly simplified version of our model, where $\lambda_0 = \mu_0 = 0$. Equation 3 gives this version of our ODE model.

$$\dot{N}_i(t) = \underbrace{\sum_j p_i\mu_j N_j(t)}_{\text{Arriving at cell i}} - \underbrace{\sum_j p_j\mu_i N_i(t)}_{\text{Departing from cell i}}, \tag{3}$$

3.3 Implementation

In this section we present a vectorized version of Eq. 3, so that we could implement it on MATLAB [22]. We used a 4th order Runge-Kutta ODE solver, native to the platform, to do so.

We start by defining a matrix $A \in \mathbb{R}^{n \times n}$ as a parameter matrix given by $A = P \times M$. $P \in \mathbb{R}^{n \times 1}$ is a column vector containing in every ith position the probability p_i of a node choosing cell i as the next destination, and $M \in \mathbb{R}^{1 \times n}$ a row vector containing in every ith position the rate μ_i at which nodes choose to leave cell i. The components of matrix A, resulting from this multiplication are $a_{ij} = p_i\ \mu_j$.

Thus, it is possible to write Eq. 3 for $\dot{N}(t) \in \mathbb{R}^{n \times 1}$ in its equivalent vectorized form as follows:

$$\dot{N}(t) = \underbrace{A \times N(t)}_{\text{Arriving}} - \underbrace{\left((A^T \times \mathbf{1}) \cdot N(t)\right)}_{\text{Departing}}, \tag{4}$$

where A^T is the transpose of matrix A, that we multiply by $\mathbf{1} \in \mathbb{R}^{n \times 1}$, a column vector of ones, to give us a resulting $n \times 1$ column vector in which every component i represents the summation of all the components of the ith row of matrix A^T. After that, we perform a component wise multiplication with the state vector $N(t)$, which gives us the number of nodes departing from a given cell. That represents the second summation in the right-handed side of Eq. 3.

4 Spatial node density of human mobility

We validate our model using real mobility traces; in other words, we show how the model can be applied to describe

Table 1 Summary of the GPS traces studied

Trace	# users	Duration (s)	Samples (s)
Quinta [2]	97	900	1
KAIST [29]	78	5,000	10
Statefair [29]	19	8,000	10

the steady-state behavior of spatial node density associated with human mobility. Three real GPS traces were used in our validation. These traces were collected in scenarios that are quite diverse, namely a city park, a university campus, and a state fair. We describe these traces in detail below as well as how we use information from the traces to estimate the parameters of our ODE framework.

4.1 Mobility traces

Table 1 summarizes the GPS traces in terms of number of users, duration of the trace, and GPS sampling period.

Quinta, refers to the "Quinta da Boa Vista Park" trace, first presented in [2]. It is a GPS trace collected at a park in the city of Rio de Janeiro, Brazil. The park has many trees, lakes, caves, and trails. It houses the National Museum of Natural History and the city Zoo. The *KAIST* trace [29], on the other hand, is a GPS trace collected at the KAIST University campus in Daejeon, South Korea. The *Statefair* trace, also available at [29], is yet another mobility scenario showing daily GPS track logs collected from the NC State Fair held in North Carolina, USA.

We select sections of the raw traces where no discontinuity occurred, i.e., we use only nodes which recorded a continuous sequence of GPS fixes that were 900, 5,000 and 8,000 s long for the Quinta, KAIST, and Statefair traces, respectively. These times were the total duration of the traces.

4.2 Parameter estimation

We extracted from the traces the distributions of *speed*, *pause time*, and *node density*. We used the trace's sampling period, for example, in the *Quinta* trace, the sampling period is $T = 1$ s. Node speed is defined as $\frac{d}{\Delta t}$ where d is the distance traveled between two consecutive entries in the GPS trace at times t_1 and t_2 and $\Delta t = t_2 - t_1$. Pause time is defined as $P = \Delta t$, if $d < threshold$, or zero otherwise. The threshold is used to account for GPS error. We set this threshold to be 2 m for KAIST and Statefair traces and 0.5 m for the Quinta trace, due to jitter in GPS update frequency.

To extract spatial node density, the area covered in the trace is divided into squared cells of 140 x 140 m. The choice of cell size was based on empirical observations, i.e., we picked a cell size that provided both adequate resolution as well as clustering. An alternate approach could be based on identifying "attraction zones", as was done in [20]. This is one of the topics of future work we plan to address. At the limit, i.e., where the cell is either infinitesimal (lower limit) or the size of the whole area (upper limit), all the traces and synthetic mobility regimes would have the same relative spatial density, namely one or zero nodes per cell for the lower limit and all the nodes in the same (unique) cell for the upper limit.

After dividing the area into cells, we took a snapshot of the number of nodes at every cell every T s. The value of $T = 10$ was used since, for the size of the cells and the speeds sampled from the traces, a node could not on average change between more than two cells during T. For every cell, at every interval T we counted the number of nodes in each cell. We then averaged the number of nodes in each cell over the course of the whole duration of the trace. The result is what we refer to as *Intensity Map (IM)* which we use to estimate the probability that a node will choose a given cell as its next destination.

In the case of real mobility, e.g., as described by GPS traces, we set the *probabilities of choosing a given cell*, p_i of our ODE model to be the normalized value of the IM for cell i, such that $p_i = \frac{IM(i)}{\sum_j IM(j)}$, where IM($i$) is the intensity in cell i.

The rate μ_i, as mentioned before, is computed as the inverse of the average time spent by the nodes in cell i. This time has two components. The time spent by the node moving towards or from a given point in the cell, and the time spent in pause at this point, which reflect both main basic parameters of human mobility, speed and pause time. This two components were empirically measured from the GPS traces and used to compute μ_i.

4.3 Results

As highlighted in previous sections, the goal of our model is to describe the steady-state behavior of spatial node density in *waypoint*-like mobility regimes in which: (1) a node chooses its next destination following some given probability distribution, (2) moves to that destination, (3) pauses for some time, and (4) repeats from step (1).

Spatial node density is defined as the percentage of cells containing $\geq k$ nodes. It can also be expressed as the probability of finding a cell containing $\geq k$ nodes. It describes the degree of "clustering" exhibited by mobility regimes and can be used to evaluate how close to reality a given synthetic mobility regime is as far as its ability to mimic the degree of clustering exhibited by real mobility.

We followed the guidelines presented in Sect. 4.2 to estimate the parameters of our model for each of the traces studied. Figures 2, 3 and 4 plot spatial node density in the Quinta, KAIST, and Statefair scenarios, respectively. Each

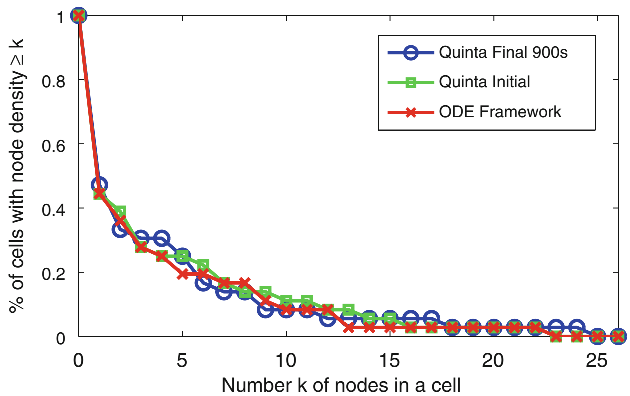

Fig. 2 Initial and final spatial node density distribution for the Quinta trace, and the respective steady-state density distribution using the proposed ODE framework

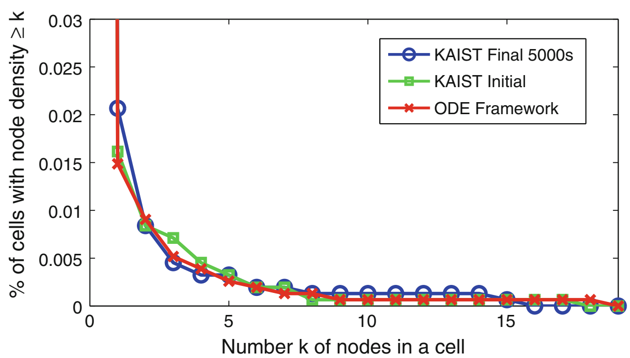

Fig. 3 Initial and final spatial node density distribution for the KAIST trace, and the respective steady-state density distribution using the proposed ODE framework

figure shows three curves plotting the spatial density: (1) at the beginning of the trace, (2) at the end of the trace, i.e., at 900 s for the Quinta Trace, 5,000 s for KAIST, and 8,000 s for Statefair, and (3) by applying our ODE framework. Note that the plots for the *KAIST* and *Statefair* traces are zoomed into the region of interest. In those two plots, the only point not shown is $k = 0$, where the percentage of cells containing 0 or more nodes $P[k \geq 0]$ is the same for every curve and it is, of course, equal to 100 %.

The *largest* deviation of our ODE model from the final density distribution measured from the traces, for any value of k at any instant is 5.36, 0.58 and 7.52 %; the average deviation from the initial distribution measured in all the instants for all values of k is 1.45, 0.06 and 2.02 % for the Quinta, KAIST and Statefair traces respectively.

5 Node density in synthetic waypoint mobility

Here we show our framework's ability to closely describe the steady-state behavior of spatial node density resulting

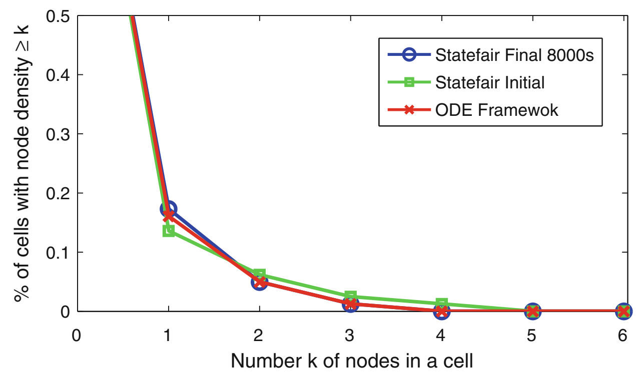

Fig. 4 Initial and final spatial node density distribution for the Quinta trace, and the respective steady-state density distribution using the proposed ODE framework

from synthetic *waypoint*-like mobility. We apply our model to two different regimes, namely Random Waypoint (RWP) [6] and the Natural [8] mobility. We start by briefly describing these two mobility regimes and then present the setup we used to generate mobility traces under them. Subsequently, we explain how we estimated the parameters for the model and present results comparing spatial node density distributions resulting from the synthetic mobility regimes and our model.

5.1 RWP mobility

Random Waypoint (RWP) mobility, an example of waypoint-like mobility regime, has been widely used in the study of multi-hop ad-hoc wireless networks (MANETs). Under RWP mobility, mobile nodes are initially placed in the area being simulated according to a given distribution. Typically, a uniform distribution is used. Each node remains in its position for a given period of time, called *pause time* P uniformly chosen in the interval $[0, P_{max}]$, where P_{max} is a pre-specified parameter. After this period, the mobile node chooses a new destination uniformly distributed in the simulation area, and a speed, also uniformly distributed in the interval $[v_{min}, v_{max}]$, where both v_{min} and v_{max} are pre-specified parameters. Once the destination is reached, the node pauses again and chooses another destination and speed, as described above.

5.2 The natural mobility regime

We also compare our results against mobility regimes that follow the *preferential attachment* principle. As representative of this family of mobility regimes, we use the Natural mobility model, or simply *Natural* [8].

As discussed in Sect. 2, *Natural* is based on attraction points, where the attractiveness of each point is proportional to the *attractor*'s popularity given by the number of nodes at or going towards it and inversely proportional to the distance to it. Thus, the probability $\Pi(a_i)$ that a node z_k chooses an attractor a_i among all possible attractors is proportional to the portion of the total attractiveness it carries: $\Pi(a_i) = \frac{\mathcal{A}_{a_i,z_k}}{\sum_j \mathcal{A}_{a_j,z_k}}$. The attractiveness of an attractor is then defined as:

$$\mathcal{A}_{a_i,z_k} = \frac{(1 + \sum_{z_j \in \mathbb{Z}, z_j \neq z_k} B(a_i, z_k))}{\sqrt{(X_{a_i} - X_{z_k})^2 + (Y_{a_i} - Y_{z_k})^2}} \tag{5}$$

where $B(a_i, z_k)$ is a Bernoulli variable, with $B = 1$ if the individual z_k is going toward or staying at attractor a_i and 0 otherwise, and X and Y are the coordinates of a node and an attractor. In our implementation, we divided the simulation area in equally sized squares, or cells, and consider each cell to be an attraction point. The coordinates (X_{a_i}, Y_{a_i}) mark the center of the i-th attraction point. Once the new destination is known, the node travels towards it with a speed that is uniformly distributed in the interval [v_{min}, v_{max}]. A pause time is randomly selected once arriving at the destination before choosing another destination and beginning the process again.

5.3 Generating synthetic waypoint mobility traces

Using a modified version of the Scengen [32] scenario simulator we generated mobility traces according to the RWP and the *Natural* mobility regimes. We setup the simulations trying to mimic the three real scenarios described in this paper, for Quinta, KAIST and Statefair. Three sets of synthetic traces were generated using the RWP and Natural mobility models. The speed range was set in a way that the average speed would match the ones measured in the GPS traces.

In order to address the decaying speed problem reported in [33], we followed the recommendations mentioned in that work. The speed range was thus set to be ± the standard deviation measured in the real traces around the measured average speed. Thus, the speed was chosen *uniformly* in a range in which the lower limit was greater than zero and where the mean matches the one measured in the real traces. This is the simplest though not the optimal solution mentioned in [33]. However, since the focus of our work is not network performance evaluation itself, we found this solution to be suitable for our purposes.

Pause time was chosen uniformly in the range [0, P_{max}], where the value of P_{max} was set to an appropriate value, in a way that the average pause time would match the one measured in the real traces. The same was done for the dimensions of the rectangular simulation area, set to be the same as in the GPS traces. Moreover, in all simulation scenarios, we used the same initial positions found in the respective real traces for the same number of users. For further discussions on the actual distributions for these traces' mobility parameters, please refer to [2, 29].

Table 2 Simulation parameters

Parameter	Quinta	KAIST	Statefair
Avg. speed (±σ)(m/s)	1.2 (±0.53)	0.72 (±0.68)	0.48 (±0.39)
Avg. pause (s)	3.6	86	72
Area (meters × meters)	840 × 840	5,000 × 5,000	1,260 × 1,260
Duration (s)	900	5,000	8,000
# nodes	97	78	19

When applying the ODE framework to describe spatial density behavior in synthetic mobility, p_i follows the probability distribution particular to the specific mobility regime used. For example, in RWP mobility, p_i is the same for every value of i, since the probability of choosing the next waypoint follows a uniform distribution. For Natural, the probability of choosing a given cell is computed "on-the-fly", based on the cell's attractiveness, defined by Eq. 5.

The rate μ_i is computed as the inverse of the average time spent by the nodes in cell i. This time has two components. The time spent by the node moving towards or from a given point in the cell, and the time spent in pause at this point. This two components were empirically measured from long simulations (10^5 s), using the same parameters for each scenario, and used to compute μ_i. A more generic approach to determine the value of this parameters for a given generic scenario is the subject of our future work.

Reported simulation results on density, comparing our ODE framework with simulated traces, reflect *10 runs* of the simulations using each mobility regime at each scenario. Table 2 summarizes the simulation parameters.

5.4 Results

Figures 5 and 6 show the results for spatial node density distribution at the Quinta scenario, for the RWP and Natural mobility regimes respectively. The plots show three curves corresponding to: (1) the initial density distribution taken from the trace and used to feed all the simulations for both mobility models, (2) the steady-state density distribution using the proposed ODE framework applied to the RWP and Natural mobility regimes, and (3) the final density distribution measured and averaged at the end of the simulations for the synthetic mobility regimes.

The first obvious observation analyzing these plots is that the synthetic mobility regimes are unable to follow the long tail behavior of the density metric presented by the

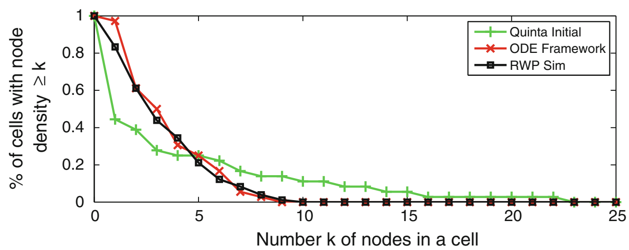

Fig. 5 Initial spatial node density distribution for the Quinta trace, simulated final density distribution using the RWP mobility regime and the respective steady-state density distribution using the proposed ODE framework applied to the RWP mobility regime

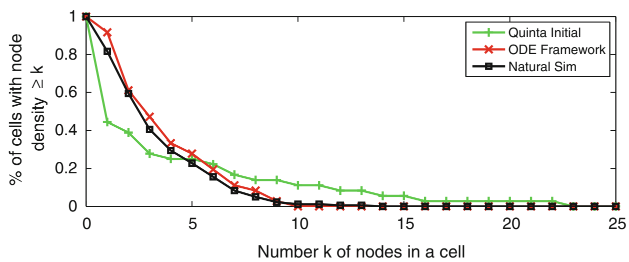

Fig. 6 Initial spatial node density distribution for the Quinta trace, simulated final density distribution using the Natural mobility regime and the respective steady-state density distribution using the proposed ODE framework applied to the Natural mobility regime

distribution measured in the real traces. Moreover, as we demonstrated in the previous section, the presented ODE model is able to follow this long tail characteristic when applying the parameters extracted from the real traces. In the case of the synthetic models, when applying the exit rate and probability of choosing the next cell, characteristic to each mobility regime, the proposed framework now behaves as the synthetic models do and is able to describe density distribution curves very similar to the RWP-like regimes studied.

Figures 7 and 8 show the results for spatial node density distributions at the KAIST scenario, for the RWP and Natural mobility regimes respectively. Similar results can be observed in Figs. 9 and 10 for the Statefair scenario. Once again, the plots for the *KAIST* and *Statefair* traces are zoomed in to the region of interest.

These show not only the flexibility of the proposed ODE framework and that it is able to describe the steady-state behavior of the density distribution of real and synthetic (RWP-like) mobility, but it also show the accuracy of the proposed approach.

5.5 Application

In addition to its applications in the study of the steady-state behavior of spatial node density of waypoint-like

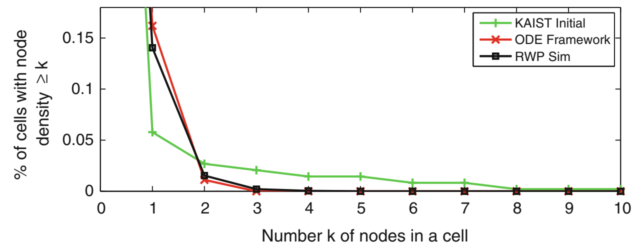

Fig. 7 Initial spatial node density distribution for the KAIST trace, simulated final density distribution using the RWP mobility regime and the respective steady-state density distribution using the proposed ODE framework applied to the RWP mobility regime

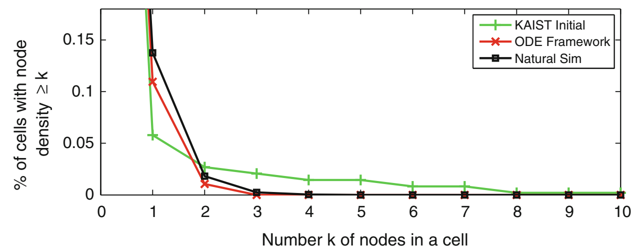

Fig. 8 Initial spatial node density distribution for the KAIST trace, simulated final density distribution using the Natural mobility regime and the respective steady-state density distribution using the proposed ODE framework applied to the Natural mobility regime

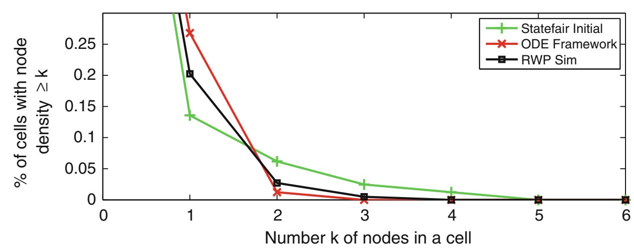

Fig. 9 Initial spatial node density distribution for the Statefair trace, simulated final density distribution using the RWP mobility regime and the respective steady-state density distribution using the proposed ODE framework applied to the RWP mobility regime

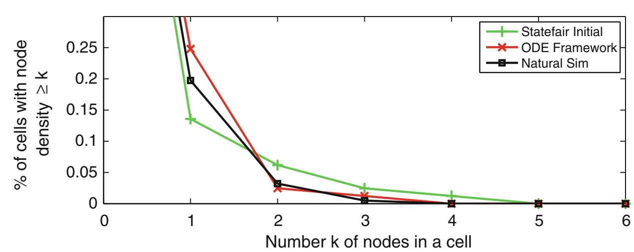

Fig. 10 Initial spatial node density distribution for the Statefair trace, simulated final density distribution using the Natural mobility regime and the respective steady-state density distribution using the proposed ODE framework applied to the Natural mobility regime

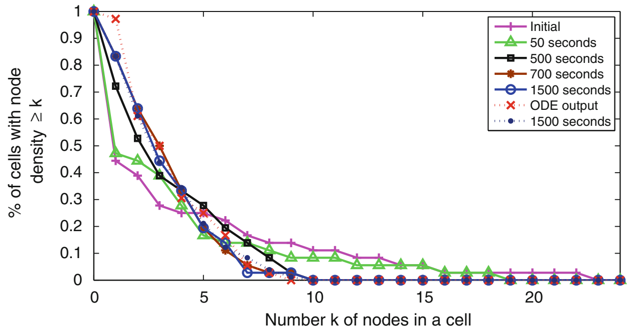

Fig. 11 RWP simulation using initial density conditions from the trace (*solid lines*), and simulation using the output of the ODE framework as the initial density conditions (*dashed lines*)

mobility regimes, we find another very interesting application of our framework. Recall that the output of our model is a vector where each position contains the expected number of nodes encountered in the cell corresponding to that position. This vector provides node density distribution at steady-state. Consequently, we can interpret the normalized value in each cell as the probability of placing a node in that cell. In other words, the normalized output vector can be seen as the steady-state spatial distribution for the waypoint-based mobility regime of interest. In this context, following the steady-state distribution given by our model, it is possible to use a *Fitness Proportionate Selection* scheme, such as the *Roulette-Wheel* described in [27] and commonly used in genetic algorithms [28] to perform initial node placement when setting up wireless network simulations.

Figure 11 illustrates this usage of our model. The solid lines in this plot reflect one simulation of the RWP regime in the Quinta scenario at instants 0 s (initial placement using the initial positions from the Quinta trace), 50, 500, 700 and 1,500 s of simulation. It is possible to see how the solid lines "move away" from the initial conditions and converge to the steady-state. Another simulation with the same seed and parameters was run, using the initial node placement given by the normalized output of the ODE framework. The dashed lines correspond to this initial distribution, and the measured final distribution at 1500 s of the same simulation run. As we can see, these two curves are very similar, showing that the output of our model gives an accurate steady-state distribution for the studied mobility regime, where the final distribution measured at the end of the simulation does not deviate form the initial placement provided by the ODE model. These two curves (the dashed ones) are a very close match also to the final distribution of the simulation that uses the trace's initial node placement (solid line at 1,500 s). Density results of simulations using the Natural mobility regime were also generated, with the exact same parameters and scenario. The same behavior was observed.

In conclusion, the main advantages of using the output of our framework for initial node placement in wireless network simulations are twofold. First, as illustrated by Fig. 11, by defining what the steady-state node density distribution is, the model saves considerable simulation time, which is the time it takes to get to the steady-state behavior. For instance, in the case of the RWP simulation, for this particular scenario, steady state is achieved after 700 s, since the 700 s curve and the 1,500 s are almost indistinguishable. Second, this result is important, once it is very difficult to determine when a model is going to reach a steady state, since this time varies from scenario to scenario and according to the mobility model and parameters used. Using our ODE model's output to establish what the steady-state behavior is constitutes a technique that is much more scientifically sound than figuring out when steady state is achieved by inspecting the mobility model's behavior over time.

6 Conclusions

In this paper, we developed a framework to study waypoint mobility regimes which have been widely used in the design and evaluation of mobile networks and their protocols. With our framework, which is based on first-order ordinary differential equations, it is possible to model the stationary behavior of spatial node density resulting from waypoint-based mobility regimes as well as real mobility described by GPS traces. We validated our approach by comparing its results against real mobility recorded by GPS traces. We also presented steady-state spatial distribution for two synthetic mobility regimes in three different scenarios. To the best of our knowledge, this is the first approach to spatial node density modeling that is generic enough that can be applied to any waypoint-like mobility regime.

We also used the proposed model to show the inability of waypoint-based mobility regimes that are based on the preferential attachment principle to maintain non-homogeneous spatial node density distributions, preserving node clusters. We showed that in steady-state, preferential attachment based models result in node density distributions that approach the distribution of a totally random mobility regime, such as the RWP mobility model.

As another application of our framework, we discussed how our modeling framework can be applied to derive stationary spatial node density distributions which can then be used to perform initial node placement when setting up mobile network simulations.

As part of our ongoing and future work plans, we plan on integrating our model to existing network simulation platforms in order to prime network simulations with steady-state node density distributions. Developing mobility regimes capable of reflecting the scale-free properties of real networks and generating mobility traces with density characteristics similar to what we measure in real mobility traces are also the focus of our on-going and future research.

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References

1. Albert, R., & Barabási, A. L. (2002). Statistical mechanics of complex networks. *Reviews of Modern Physics 74*(1), 47–97.
2. Azevedo, T., Bezerra, R., Campos, C., & De Moraes, L. F. M. (2009). An analysis of human mobility using real traces. In: Wireless Communications and Networking Conference, 2009. WCNC 2009. IEEE, pp. 1–6.
3. Balachandran, A., Voelker, G. M., Bahl, P., & Rangan, P. V. (2002). Characterizing user behavior and network performance in a public wireless lan. In: *Proceedings of the 2002 ACM SIGMETRICS international conference on Measurement and modeling of computer systems*, SIGMETRICS '02, pp. 195–205.
4. Balazinska, M., & Castro, P. (2003) Characterizing mobility and network usage in a corporate wireless local-area network. In: *Proceedings of the 1st international conference on Mobile systems, applications and services, MobiSys '03*, pp. 303–316.
5. Bettstetter, C., Resta, G., & Santi, P. (2003). The node distribution of the random waypoint mobility model for wireless ad hoc networks Mobile Computing. *IEEE Transactions on, 2*(3), 257–269.
6. Bettstetter, C., Hartenstein, H., & Pérez-Costa, X. (2004). Stochastic properties of the random waypoint mobility model. *Wireless Networks, 10*(5), 555–567.
7. Bettstetter, C., Gyarmati, M., & Schilcher, U. (2007). An inhomogeneous spatial node distribution and its stochastic properties. In: *Proceedings of the 10th ACM Symposium on Modeling, analysis, and simulation of wireless and mobile systems, MSWiM '07*.
8. Borrel, V., de Amorim, M. D., & Fdida, S. (2005). On natural mobility models. In I. Stavrakakis & M. Smirnov (Eds.), Autonomic Communication, Second International IFIP Workshop, WAC 2005, Athens, Greece, Oct 2–5, 2005, *Revised selected papers*, Springer, Lecture Notes in Computer Science, Vol. 3854 (pp. 243–253).
9. Camp, T., Boleng, J., & Davies, V. (2002). A survey of mobility models for ad hoc network research. *Wireless Communications and Mobile Computing, 2*, 483–502.
10. Chen, Z., Gao, L., & Kwiaty, K. (2003). Modeling the spread of active worms. In: *INFOCOM 2003. Twenty-second annual joint conference of the IEEE Computer and Communications*. IEEE Societies, Vol. 3, pp. 1890–1900.
11. CRAWDAD. (2013). http://crawdad.cs.dartmouth.edu/
12. Daley, D. J., & Gani, J. (1999). *Epidemic modelling*. Cambridge University Press.
13. Fawal, A. E., yves Le Boudec, J., Salamatian, K., Self, A., & Service, L. E. (2006). *Self-limiting epidemic forwarding*. Technical Report., In The First IEEE WoWMoM Workshop on autonomic and opportunistic communications—AOC2007.
14. Garetto, M., & Leonardi, E. (2007). Analysis of random mobility models with partial differential equations. *IEEE Transactions on Mobile Computing, 6*(11), 1204–1217.
15. Gyarmati, M., Schilcher, U., Brandner, G., Bettstetter, C., Chung, Y. W., & Kim, Y. H. (2008). Impact of random mobility on the inhomogeneity of spatial distributions. In: *Global Telecommunications Conference, 2008*. IEEE GLOBECOM 2008, pp. 1–5.
16. Heimlicher, S., & Salamatian, K. (2010). Globs in the primordial soup: The emergence of connected crowds in mobile wireless networks. In: *Proceedings of the eleventh ACM international symposium on mobile ad hoc networking and computing, MobiHoc '10*, pp. 91–100.
17. Henderson, T., Kotz, D., & Abyzov, I. (2004). The changing usage of a mature campus-wide wireless network. In: *Proceedings of the 10th annual international conference on Mobile computing and networking, MobiCom '04*.
18. Jardosh, A., Belding-Royer, E., Almeroth, K., & Suri, S. (2005). Real-world environment models for mobile network evaluation. *Selected areas in communications. IEEE Journal, 23*(3), 622–632.
19. Kesidis, G., Jin, Y., Mortazavi, B., & Konstantopoulos, T. (2006). Qrp03-1: An epidemiological model for file-sharing with bittorrent-like incentives: The case of a fixed peer population. In: *Global Telecommunications Conference, 2006*. GLOBECOM '06. IEEE, pp. 1–5.
20. Kim, M., Kotz, D., & Kim, S. (2006). Extracting a mobility model from real user traces. In: *INFOCOM 2006. 25th IEEE International Conference on Computer Communications, Proceedings*, pp. 1–13.
21. Lim, S., Yu, C., & Das, C. R. (2006). Clustered mobility model for scale-free wireless networks. In: *Local Computer Networks, Proceedings 2006 31st IEEE Conference on*.
22. MATLAB. (2010). *version 7.10.0 (R2010a)*. The MathWorks Inc.
23. Mei, A., & Stefa, J. (2009). *Swim: A simple model to generate small mobile worlds*. In: INFOCOM 2009, IEEE, pp 2106–2113.
24. Musolesi, M., & Mascolo, C. (2007). Designing mobility models based on social network theory. *ACM SIGMOBILE Mobile Computing and Communication Review, 11*, 59–70.
25. Musolesi, M., & Mascolo, C. (2009). Mobility models for systems evaluation. In: *Middleware for network eccentric and mobile applications*, Springer, pp. 43–62.
26. Navidi, W., & Camp, T. (2003). Stationary distributions for the random waypoint mobility model. *IEEE Transactions on Mobile Computing, 3*, 99–108.
27. Nunes, B. A., & Obraczka, K. (2011). On the invariance of spatial node density for realistic mobility modeling. In: *Mobile adhoc and sensor systems (MASS), 2011 IEEE 8th International Conference on*, pp. 322–331.
28. Pan, Z., LS, K., & Chen, Y. (2000). *Evolutionary computation*. Beijing, China: Tsinghua University Press.
29. Rhee, I., Shin, M., Hong, S., Lee, K., & Kim, S. (2009). CRAWDAD http://crawdad.cs.dartmouth.edu/ncsu/mobilitymodels/GPS/KAISTv.2009-07-23.
30. Staniford, S., Paxson, V., & Weaver, N. (2002). How to own the internet in your spare time. In: *Proceedings of the 11th USENIX security symposium*, pp. 149–167.
31. Tang, D., & Baker, M. (2000). Analysis of a local-area wireless network. In: *Proceedings of the 6th annual international conference on mobile computing and networking, MobiCom '00*, pp. 1–10.
32. The Scenario Generator. (2001). http://isis.poly.edu/qiming/scengen/index.html.
33. Yoon, J., Liu, M., & Noble, B. (2003). Random waypoint considered harmful. In: INFOCOM 2003. Twenty-second annual joint Conference of the IEEE Computer and Communications. *IEEE Societies, 2*, pp. 1312–1321, vol.2.
34. Zhang, X., Neglia, G., Kurose, J., & Towsley, D. (2007). Performance modeling of epidemic routing. *Computer Networks, 51*, 2867–2891.

35. Zou, C. C., Gong, W., & Towsley D. (2002). Code red worm propagation modeling and analysis. In: *Proceedings of the 9th ACM conference on Computer and communications security, CCS '02*, pp. 138–147.

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