Statistical Equivalent Models for Computer Simulators with an Application to the Random Waypoint Mobility Model

Kumar Viswanath
Katia Obraczka
Athanasios Kottas
Bruno Sansó
School of Engineering
University of California Santa Cruz
Santa Cruz
California
kumarv@soe.ucsc.edu

Statistical Equivalent Models, or SEMs, have recently been proposed as a general approach to study computer simulators. By fitting a statistical model to the simulator's output, SEMs provide an efficient way to quickly explore the simulator's result. In this paper, we develop a SEM for random waypoint mobility, one of the most widely used mobility models employed by network simulators in the evaluation of communication protocols for wireless multi-hop ad hoc networks (MANETs). We chose the random waypoint mobility model as a case study of SEMs due to recent results pointing out some serious drawbacks of the model (e.g., [1]). In particular, these studies show that, under the random waypoint mobility regime, average node speed tends to zero in steady state. They also show that average node speed varies considerably from the expected average value for the time scales under consideration in most simulation analysis. In order to investigate further the behavior of the random waypoint model, we developed a SEM that captured speed decay over time under random waypoint mobility using maximum speed and terrain size as input parameters. A Bayesian approach to model fitting was employed to capture the uncertainty due to unknown parameters of the statistical model. The SEM is given by the posterior predictive distributions of the average node speed as a function of time. A direct result from our model is that, by characterizing average node speed as a function of time, our approach provides an accurate estimate of the "warm-up" period required by simulations using the random waypoint mobility model. Simulation data from the "warm-up" period can then be discarded to obtain accurate protocol performance results. Given that random waypoint mobility is still, by far, the most widely used mobility model in the evaluation of MANETs, the contribution of this work is potentially significant as it allows network protocol designers to continue to use the original random waypoint mobility model and yet obtain accurate results characterizing MANET protocol performance.

Keywords: Ad-hoc networks, Gaussian process, mobility model, non-linear regression, random waypoint, statistical equivalent models(SEMs)

1. Introduction

The problem of fitting a statistical model to computer simulator results has recently been receiving some attention in the literature (e.g., [2]) as an efficient way to perform fast exploration of the simulator's output. The method, referred to as Statistical Equivalent Modeling, consists of creating a relatively simple statistical model to approximate some function of the output of a computer simulator.

SIMULATION, Vol. 00, Issue 0, Xxxxxxxx 2007 000–000

DOI: 10.1177/0037549707080610
Figures 1–11 appear in color online: http://sim.sagepub.com

The resulting Statistical Equivalent Model (SEM) must be flexible enough to capture most of the variability in the simulator's output. Since SEMs are used as surrogate models, it is important that the uncertainty in the SEM predictions be easily assessed in order to quantify its performance. SEMs are used for a variety of purposes which can be roughly classified in four groups:

(1) **Model calibration** to find ranges of the simulator input that produce sensible output; e.g. fitting a statistical model to test automotive engine data. Traditionally, lookup tables in engine controllers are calibrated directly from test data which require that engine tests be run at almost all operating points to achieve complete calibration. Using SEMs it is possible to use design-of-experiment techniques to plan reduced engine tests and then fit statistical model to the results and then use those models to perform analytical calibration.

(2) **Data assimilation** which consists of merging simulator output with observations.

(3) **Model validation** for comparison of results from a simulator to observations or other simulators; e.g. SEMs can be used to characterize traffic patterns for On-Chip Interconnection networks based on real or simulation traces which can help in Network-on-Chips design space exploration.

(4) **Model description and comparison** for describing and comparing certain configurations of the simulator. The latter is the application of Statistical Equivalent Modeling we employ in the work described herein.

Regarding the literature on SEMs, the proposed methods, typically, involve Gaussian process response-surface approximations, i.e., the use of Gaussian processes to approximate the function that represents the computer simulator output (e.g., [3]). The literature includes a number of Bayesian approaches with a range of corresponding applications (see, e.g., [4–12]). Bayesian methodology is particularly suited to addressing the four issues discussed above, to quantifying multiple sources of error and uncertainty in computer simulators, and to combining multiple sources of information. We note that there has been relatively limited work on statistical methodology for the analysis of computer experiments under the practically important setting where the computer simulator produces functional output (see [13, 14] for some recent work in this direction).

The SEM we develop in this paper (categorized as Model Description and Comparison SEM) handles functional computer simulator data, in particular, data on the average node speed, as a function of time, based on the random waypoint mobility regime. This is a mobility model that has been extensively used by packet-level network simulators to study the performance of communication protocols for multi-hop wireless ad hoc networks (MANETs).

Packet-level network simulators (e.g., ns-2 [15], GloMoSim [16], QualNet [17], OPNET [18]) have been an extremely popular platform for evaluating MANET protocols. There are clear advantages to using simulations when evaluating network (in particular, MANET) protocols, including the ability to reproduce experiments and subject protocols to a wide range of network topologies and conditions, for example mobility patterns. Topology, number of network nodes and node mobility are important parameters that can significantly affect protocol performance.

Most existing network simulators employ random waypoint mobility to model how nodes move on a terrain [19]. Nodes in the random waypoint regime move according to the following rules: (1) each node picks a destination randomly within the simulation area and also picks a speed v that is uniformly chosen between v_{min} and v_{max}. Each node then moves toward the destination over a straight line with speed v. (2) upon reaching the destination, a node pauses for some *pause-time*; (3) the node then picks the next destination and the process re-starts. Typically, the values of v_{min}, v_{max}, and *pause-time* are parameters of the simulation and are selected according to the requirements and operating environment of the application at hand.

Recently, it has been reported that the random waypoint model exhibits some originally unforeseen anomalous behavior. More specifically, it has been shown that, under the random waypoint model, the average node speed decays with time [1]. It has also been shown that nodes moving according to the random waypoint model tend to concentrate in the middle of the simulation region, resulting in non-uniform node spatial distribution. In the specific case where $v_{min} = 0$, as time $t \to \infty$, node speeds tend to zero, resulting in a stationary system. One important effect of this behavior is that, if simulations using the random waypoint model do not run for sufficiently long periods beyond the initial steep decay, the corresponding simulation results will not be accurate. In fact, variations of up to 40% in ad hoc routing performance over a 900-s simulation have been detected [1].

From the above discussion, one important consideration is how long does it take for the system to converge to steady state. Given this information, one easy "fix" to the random waypoint model is to run simulations long enough to guarantee that protocol performance evaluation is conducted after steady state is reached to guarantee accurate results. In this paper, we introduce a novel approach to study the behavior of the random waypoint regime. We develop a SEM to predict average node speed (through both point and interval estimates) as a function of input parameters v_{max} and field size. Since our SEM also characterizes average node speed as a function of time, it offers an efficient alternative to obtaining an accurate estimate of how long simulation experiments take to "warm-up".

Simulation data from the "warm-up" period can then be discarded to obtain accurate protocol performance results. Since random waypoint mobility continues to be, by far, the most widely used mobility model in the evaluation of MANETs, our model allows protocol designers to use the original random waypoint mobility model and still characterize MANET protocol performance accurately.

To build a SEM for random waypoint mobility, we consider a random waypoint simulator where the inputs are terrain size and maximum node speed. We run the simulator for a number of different configurations of these two variables. We then fit a statistical model to the resulting average node speed at different times. We validate the statistical model by comparing its predictions against the actual simulator results. We use the resulting model to measure the decay of average node speed and quantify the time that it takes for the "warm-up". We also show that our random waypoint mobility SEM is able to provide information on the warm-up period for different combinations of input parameters significantly faster than running pre-simulations of the mobility model for different input combinations. For instance, using our model, it took us 20 min to compute the point estimates of the warm-up period over a grid of values for v_{max} and field size (and for two different values of speed decay). Using the same (reasonably fast) machine, it would take approximately 65 h to run pre-simulations for the same grid of v_{max} and field size values.

In the rest of the paper we present our statistical model in detail. Section 2 puts our work in perspective by describing related efforts in modeling the random waypoint regime. In Section 3, we present the methodology employed to formulate and develop the model. Section 4 describes the proposed statistical model. In Section 5, we present results obtained from the model and evaluate its accuracy by validating it against data obtained from the simulator. Section 5.3 discusses directions for future work and in Section 6 we elaborate on other possible approaches in developing a SEM based on Gaussian process priors. Finally, Section 7 presents concluding remarks.

2. Related Work

Mobility models are an important component of network simulators and are one of the key factors affecting the performance of ad-hoc network protocols. A number of mobility models for ad-hoc networks have been proposed and evaluated [20–23]. One of the most widely used mobility models is the random waypoint model ([19, 24, 25]) described in Section 1. This model is implemented in a number of current network simulation platforms such as ns-2 [15], GloMoSim [16], and Qualnet [17].

However, it has been shown in [1] that under the random waypoint regime, the average node speed decays with time before reaching steady state and the settling time to reach steady state increases as the minimum speed parameter v_{min} of the model decreases. In particular, the default random waypoint models distributed with ns-2 and GlomoSim use $v_{min} = 0$ which causes the average node speed to steadily decrease over time. In [1], the impact of this speed decay on ad-hoc routing protocols like DSR [19] and AODV [26] was also investigated. It was shown that speed decay can result in performance variations of around 40% over simulation times typically used in the study of ad-hoc network protocols. One suggested solution was to use non-zero minimum speed or to discard results from the "burn-in" period, i.e., the simulation period during which speed decay is most dramatic.

There have been several other bodies of work such as [27–29] which have investigated the spatial node distribution for the random waypoint model.

In [30], a framework for analyzing the speed decay of mobility models was proposed; additionally, based on this framework, a technique to obtain the stationary equivalent to mobility models that exhibit the speed decay behavior was introduced. Essentially, the proposed strategy is to choose initial speeds from the stationary distribution and subsequent speeds according to the original distribution. Similarly, in [31] the authors have used palm calculus to provide necessary and sufficient conditions for a stationary regime to exist under the random waypoint model and presented an algorithm to start simulations in the steady state (so called perfect simulation).

The main difference of our work compared to other approaches is that we propose a novel method to study the behavior of the random waypoint model, which uses a statistical model to characterize speed decay. Our model is able to predict average node speed (through both point and interval estimates) as a function of input parameters v_{max} and field size. Our model also offers an efficient alternative to obtaining accurate results from simulations using the original Random Waypoint model.[1] More specifically, as it will become clear in Section 5.2, using our statistical model, one can obtain the speed decay as a function of time (as well as the input parameters). This allows protocol designers running simulations to plan their experiments accordingly so as to discard results from the "warm-up" period and hence perform accurate protocol performance evaluation.

3. Methodology

We used GloMoSim [16] as the simulation platform for the initial mobility experiments. The simulation setup consisted of 150 nodes moving according to the random waypoint model with v_{max} from the set {2, 3, 4, 5, 7.5, 10, 12.5, 15, 17.5, 20} m/s and $v_{min} = 0$. The pause-time was set to 0 for all experiments. The field-size was varied in the range {500, 1000, 1500, 2000, 2500, 3000} m^2.

1. Note that the alternative is to run pre-simulations of the mobility model for different combinations of parameters of interest.

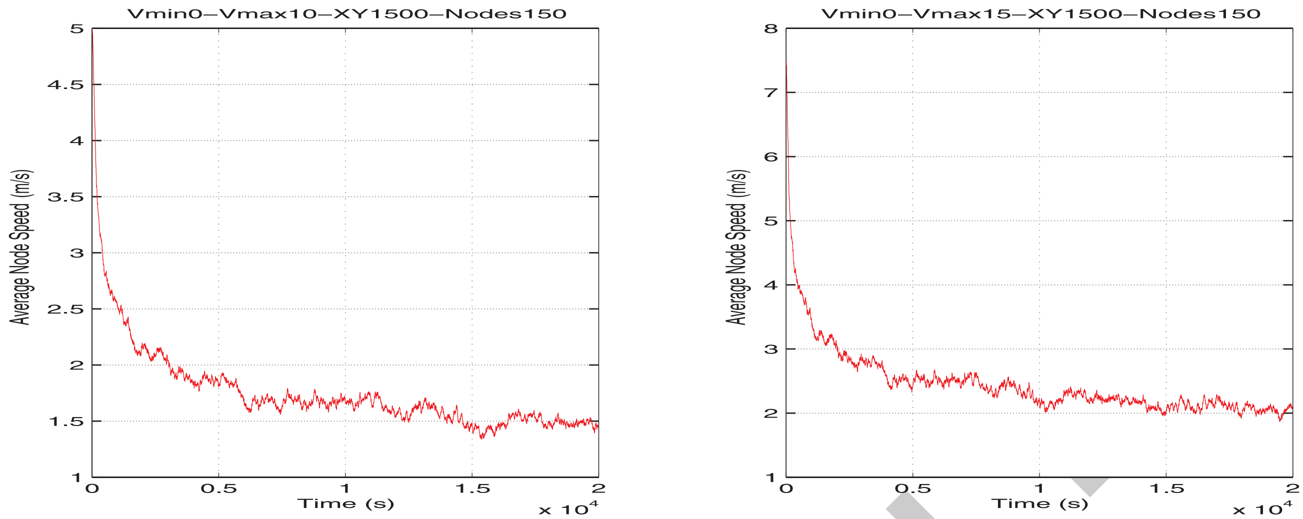


Figure 1. Speed decay under the random waypoint model

Hence we ran mobility simulations for 60 different combinations of v_{max} and field-size with each run averaged over 10 different seed values. The total duration of the mobility experiments was set to 20,000 s and we captured the average node speed as reported by the simulator every 5 s. As noted previously the values were averaged over 10 different runs using different seed values. The data obtained from these mobility experiments was used as "computer simulator data" for our statistical model.

One key observation that helped to simplify model formulation was the implicit relationship between terrain size and number of nodes used in simulation experiments. For a given transmission range, the terrain size chosen normally dictates the minimum number of nodes required to ensure that the network is connected.[2] Hence our model implicitly accounts for number of nodes through the "field-size" parameter, which is defined as the two-dimensional region within which nodes can move.

Figure 1 is a pictorial representation of the speed decay suffered by nodes using the Random Waypoint mobility model. Note that, the average initial speed of the nodes is $(v_{max} - v_{min})/2$ as expected and then starts decaying with time. This is similar to the results observed in [1].

4. Statistical Model

We develop a statistical model for average node speed, using computer simulator data obtained, as discussed in Section 3, for 10 different choices of v_{max} and six different field sizes. In what follows we use the notation v for v_{max}, f for field-size, t for time in s, and y_t for the average node speed at time t. First, we construct the non-linear regression model,

$$y_t = \frac{c}{\{1 + b(t/1000)\}^a} + \varepsilon, \tag{1}$$

to capture speed decay as a function of time for specified v_{max} and field-size values. Here, ε denotes the error term, and a, b, c are the parameters that control the shape of the non-linear regression curve. We fitted these models (for each of the 60 combinations of v_{max} and field-size values) using least squares and obtained a set of 60 triplets corresponding to the fitted values of a, b and c. By exploring the dependence of these values on v and f, we generalize model (1) making the coefficients a, b and c dependent on v and f. Hence, we obtain a SEM for the average node speed, corresponding to any combination of v and f, given by $y_t(v, f) = g(t, v, f; \mathbf{a}, \mathbf{b}, \mathbf{c}) + \varepsilon$, where

$$g(t, v, f; \mathbf{a}, \mathbf{b}, \mathbf{c}) = \frac{c(v, f)}{\{1 + b(v, f)(t/1000)\}^{a(v,f)}} \tag{2}$$

with

$$\begin{aligned} a(v, f) &= \exp\{a_1 + a_2 \log(f/v) + a_3 \log(\log(f/v)) \\ &+ a_4 \log(\log((v/f) + 1)) \\ &+ a_5 \log(\log(v + 0.5))\} \\ b(v, f) &= \exp\{b_1 + b_2 \log v + b_3 \log f \\ &+ b_4 \log(\log(f/v)) + b_5 \log(\log f) \\ &+ b_6 \log(\log(v + 0.5))\} \\ c(v, f) &= \exp\{c_1 + c_2 \log v + c_3 \log f\}. \end{aligned} \tag{3}$$

2. Node mobility can cause the network to be disconnected at certain times

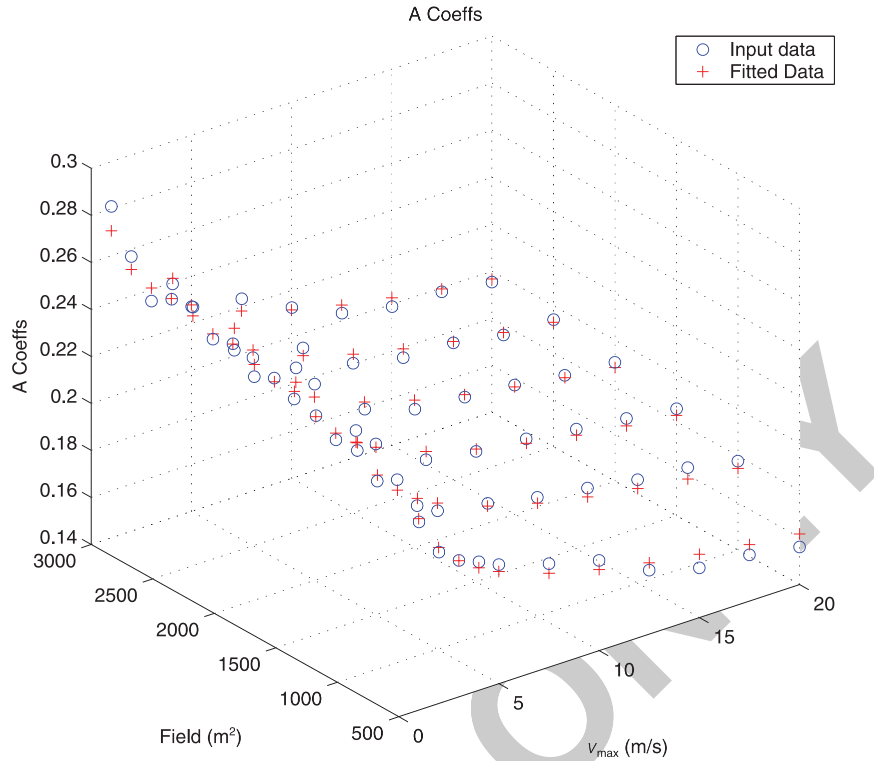


Figure 2. NLS fit for A Coeffs

Here $\mathbf{a} = (a_1, \ldots, a_5)$, $\mathbf{b} = (b_1, \ldots, b_6)$ and $\mathbf{c} = (c_1, c_2, c_3)$ denote the vectors of the unknown coefficients. These can be estimated from the computer simulator data. We note that the seemingly complicated structure of the proposed statistical model was obtained using a constructive approach. First, working with various combinations of v_{max} and field-size, we observed that a non-linear regression model with, at least, three parameters was required to model average node speed as a function of time. The form in (1) emerged as a particularly successful one in capturing the shape of these functions. Next, to obtain the expressions for $a(v, f)$, $b(v, f)$, and $c(v, f)$ in (4.2), we explored several different alternative formulations, preferring the ones given above, since they provided the best fit with the smallest number of parameters. Figures 2 to 4 illustrate the non-linear least square fits for $a(v, f)$, $b(v, f)$, and $c(v, f)$ from the original simulator data.

Of course, different statistical models could be developed by using different forms for $a(v, f)$, $b(v, f)$, and $c(v, f)$ in (2), or, perhaps, a different non-linear regression model. However, based on our extensive empirical study of the computer simulator data, we would argue that, within a parametric framework, a significant reduction in the number of parameters is not possible without sacrificing flexibility of the resulting SEM. More general approaches to the development of the SEM are discussed in Section 6.

A critical advantage of this formulation is that, once the 14 unknown parameters are estimated, one can estimate the average node speed for any combination of field-size and v_{max}, and for any time.

The estimation of the parameters in the SEM was performed by assuming that the error term follows a normal distribution with zero mean and variance σ^2. Therefore, given the data $Y = \{y_t(v_i, f_j); t = 1, \ldots, T; i = 1, \ldots, 10; j = 1, \ldots, 6\}$, we obtain the likelihood for the parameter vector, which is denoted by $\theta = (\mathbf{a}, \mathbf{b}, \mathbf{c}, \sigma^2)$, as

$$\begin{aligned} L(\theta|Y) &= \prod_{t,i,j} (2\pi\sigma^2)^{-1/2} \exp\{-\frac{1}{2\sigma^2}(y_t(v_i, f_j) \\ &- g(t, v_i, f_j; \mathbf{a}, \mathbf{b}, \mathbf{c}))^2\}. \end{aligned} \tag{4}$$

We estimate θ using a Bayesian approach. This is based on exploring the posterior distribution $p(\theta|Y)$. We consider a non-informative prior $p(\mathbf{a}, \mathbf{b}, \mathbf{c}, \sigma^2) \propto 1/\sigma^2$. Thus $p(\theta|Y) \propto 1/\sigma^2 L(\theta|Y)$. Under squared error loss, the optimal estimator is given by the posterior expectation $E(\theta|Y)$.

Given the difficulties involved in describing, integrating or maximizing $p(\theta|Y)$, which is a 15-dimensional function, we resort to Markov Chain Monte Carlo (MCMC) methods to obtain samples from $p(\theta|Y)$. The

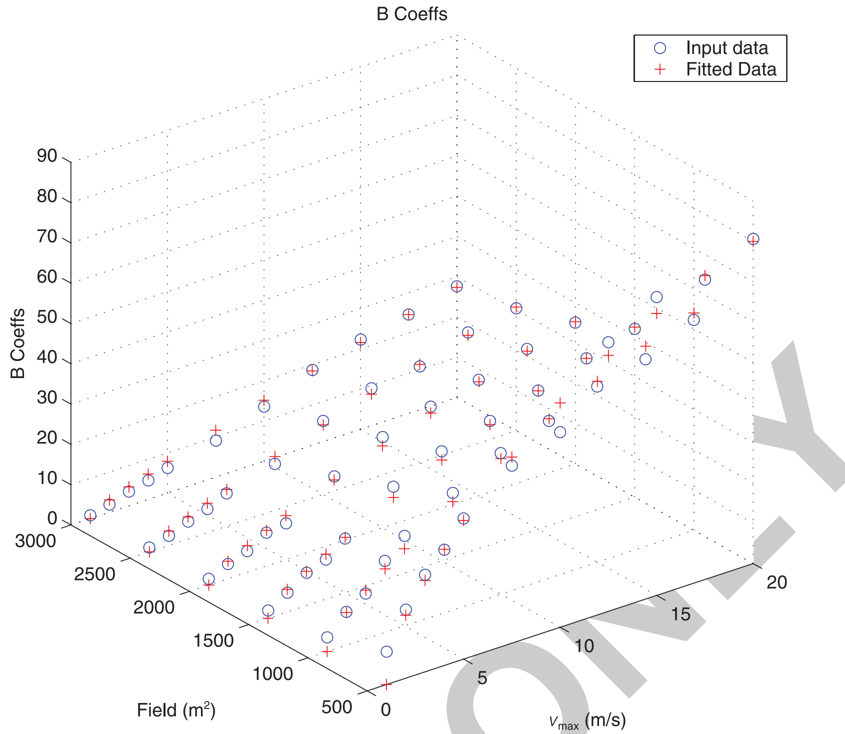


Figure 3. NLS fit for B Coeffs

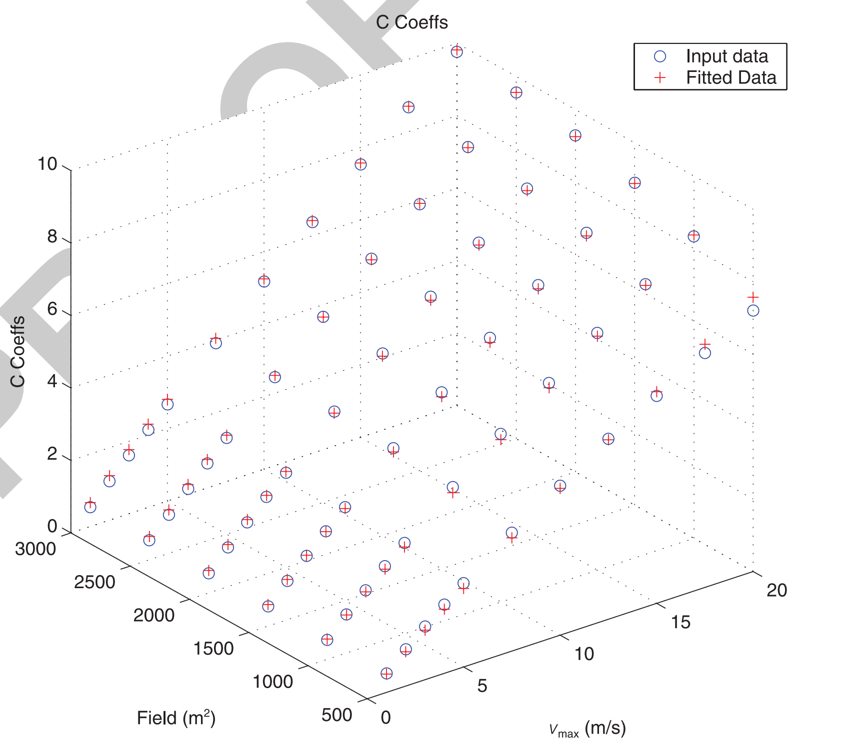


Figure 4. NLS fit for C Coeffs

idea of MCMC methodology is to construct a Markov chain that is easy to sample from and such that its equilibrium distribution is $p(\theta|Y)$ [32]. Before we describe the Markov chain that we used, we note that $p(\mathbf{a}, \mathbf{b}, \mathbf{c}, \sigma^2|Y) = p(\sigma^2|\mathbf{a}, \mathbf{b}, \mathbf{c}, Y)p(\mathbf{a}, \mathbf{b}, \mathbf{c}|Y)$, where

$$p(\mathbf{a}, \mathbf{b}, \mathbf{c}|Y) \propto A^{-60T/2}$$

$$p(\sigma^2|\mathbf{a}, \mathbf{b}, \mathbf{c}, Y) \propto (\sigma^2)^{-(60T+2)/2} \exp\{-A/(2\sigma^2)\},$$

with $A = \sum_{t,i,j}(y_t(v_i, f_j) - g(t, v_i, f_j; \mathbf{a}, \mathbf{b}, \mathbf{c}))^2$. Thus we recognize $p(\sigma^2|\mathbf{a}, \mathbf{b}, \mathbf{c}, Y)$ as the density of an inverse gamma distribution with shape $60T/2$ and scale $A/2$.

To obtain samples from the posterior $p(\theta|y)$ we follow the steps:

(1) Set initial values θ_0 and total number of iterations K

(2) Loop for $k = 1, \ldots, K$

(3) At iteration k, denote the current samples with the super-index k, and sample a vector of candidates $(\mathbf{a}^*, \mathbf{b}^*, \mathbf{c}^*)$ from a normal distribution with mean $(\mathbf{a}^k, \mathbf{b}^k, \mathbf{c}^k)$ and covariance matrix $\mathbf{V}$.

(4) Calculate $\alpha = \min\{1, r\}$ where

$$r = \frac{p(\mathbf{a}^*, \mathbf{b}^*, \mathbf{c}^*|Y)}{p(\mathbf{a}^k, \mathbf{b}^k, \mathbf{c}^k|Y)}.$$

(5) Sample u from a uniform distribution on (0,1).

(6) If $u < \alpha$ then sample $(\sigma^2)^*$ from an inverse gamma distribution with shape $60T/2$ and scale $A^*/2$, where A^* denotes the evaluation of A at the candidate values $(\mathbf{a}^*, \mathbf{b}^*, \mathbf{c}^*)$. Let $(\mathbf{a}^{k+1}, \mathbf{b}^{k+1}, \mathbf{c}^{k+1}) = (\mathbf{a}^*, \mathbf{b}^*, \mathbf{c}^*)$, $(\sigma^2)^{k+1} = (\sigma^2)^*$ and cycle.

(7) If $u > \alpha$, let $(\mathbf{a}^{k+1}, \mathbf{b}^{k+1}, \mathbf{c}^{k+1}) = (\mathbf{a}^k, \mathbf{b}^k, \mathbf{c}^k)$, $(\sigma^2)^{k+1} = (\sigma^2)^k$ and cycle.

After an initial burn-in period, the results from this chain yield a sequence of samples θ^k whose distribution is approximately $p(\theta|Y)$. These posterior samples can be used to obtain inference for θ.

5. Results and Discussion

In this section we present results obtained from the statistical model and assess its performance using mobility data obtained from the simulator. As explained in Section 4, we ran a MCMC algorithm in MATLAB to obtain samples from the posterior distribution for $p(\theta|Y)$. These samples were then used to estimate $a(v, f)$, $b(v, f)$ and $c(v, f)$ for different combinations of v_{max} and field-size. The estimates thus obtained were then used to evaluate for each combination of (v, f) the posterior mean of y_t as given in equation (2) for 4000 time-points up to 20,000 s. Note that for each combination of (v, f) we obtain samples from the entire posterior distribution for equation (2). We present both point estimates and interval estimates (denoted by dashed lines in the subsequent figures) based on 5 and 95% quantiles of the posterior samples.

Figure 5 depicts the comparison between the simulator data and posterior point and interval estimates based on the statistical model for v_{max} = 2 m/s, while Figures 6 to 8 show the comparison for v_{max} = 10, 12.5 and 20 m/s, respectively. Note that, in the figures we only present model fits up to 15,000 s for the sake of clarity as the behavior beyond 15,000 s is very similar.

As seen from these figures, the statistical model produces good fits as compared to the mobility data from the simulator. The interval estimates tend to capture the variability of the original data as well. One minor discrepancy is the tendency of the statistical model to overestimate the actual values of the average node speed at t close to 0.

5.1 Model Validation

In order to verify the accuracy of the proposed estimator we also ran some validation tests. In these tests we used the random waypoint mobility SEM of Section 4 to estimate average node speed over time for values of v_{max} that are not included in the set of 10 values used in developing the SEM. We then validate the SEM predictions against computer simulator data obtained under the new v_{max} values. In particular, we used two different values of v_{max}, i.e., {8, 25}, keeping all other simulator parameters constant. Note that one of the v_{max} values, i.e., 8 m/s is within the data range originally considered while the other value, i.e., 25 m/s is outside the data range used to formulate the SEM. Figures 9 and 10 illustrate that the statistical model provides good fits for the new simulator data as well.

5.2 Discussion

As mentioned in Section 1, the main contribution of this work is the ability of the statistical model to predict the average node speed (through both point and interval estimates) as a function of input parameters v_{max} and field-size. One of the recommended techniques to obtain accurate simulation results using the random waypoint model is to discard results from the "warm-up" period during which average node speed is still decaying. The proposed statistical model is useful in providing inference for the "warm-up" period for a specific simulation using the following equation

$$t_{warm-up} = 1000.b^{-1}\{(c/y_t)^{a^{-1}} - 1\},$$

where y_t is the required value for the speed decay and a, b and c are functions of v_{max} and field-size as defined in

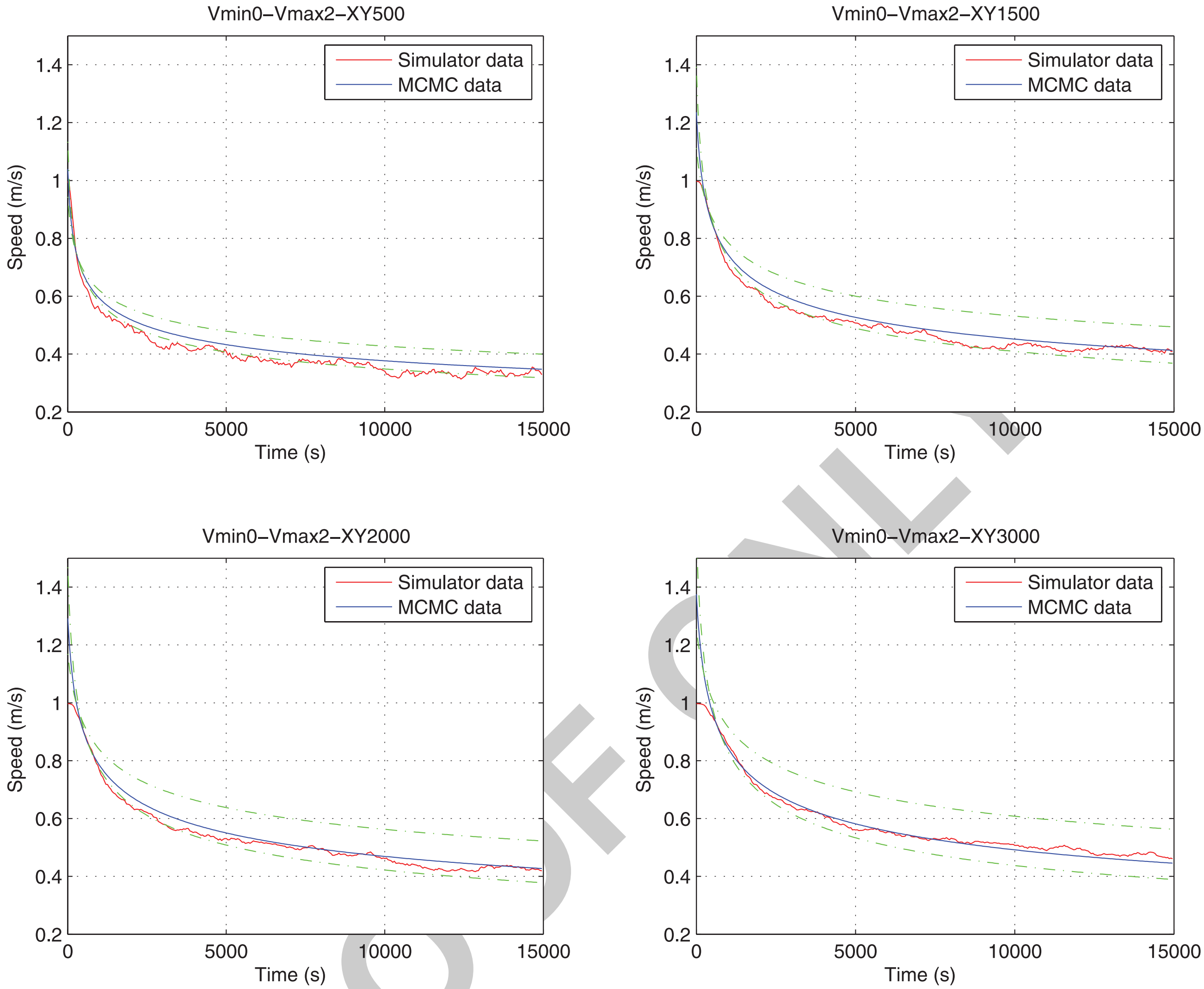


Figure 5. Inference: v_{max} 2 m/s

Section 4. Hence we can obtain the entire posterior distribution for $t_{warm-up}$ as a function of v_{max} and field-size for different values of speed decay y_t.

Figure 11 represents the point estimates of the "warm-up" period for a grid (of size 1250) over a range of commonly used combinations of v_{max} and field-size for two different values of y_t.

To put these results in perspective, the alternative approach would require running pre-simulations of the mobility model. For the 1250 different combinations of v_{max} and field-size considered above this would take approximately 65 h for 10 different seed values on a sufficiently fast simulation machine, whereas our approach required approximately 20 min of computing time.

5.3 Extensions

As an initial exercise in capturing the speed decay of the random waypoint mobility model we have made the simplifying assumption of setting the pause-time to zero as this does not affect the speed-decay behavior. This is also corroborated by the authors in [1]. However, it is possible to extend the statistical model to include pause-time as another parameter. More specifically, the effect of the pause-time can be captured by an additional parameter in equation (2).

Another area of future work involves extending the random waypoint mobility SEM to incorporate non-zero minimum speeds ($v_{min} \neq 0$).

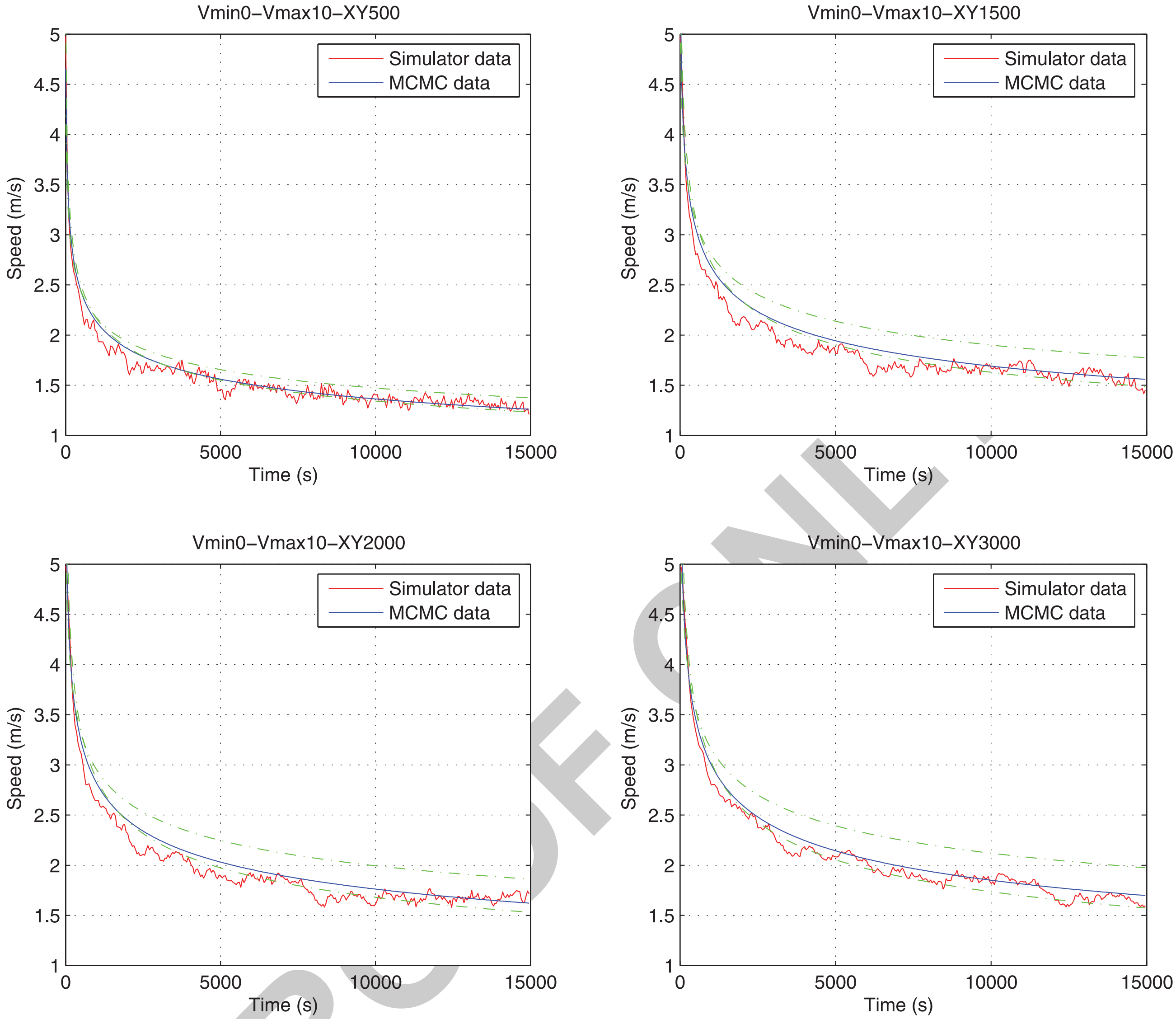


Figure 6. Inference: v_{max} 10 m/s

6. Elaborations of the SEM Framework

In this section, we consider extensions, in two different directions, of the statistical modeling framework presented in Section 4. Although these extensions are discussed in the context of the random waypoint model, they serve to indicate generic methodology for the analysis of output from computer simulators, based on Gaussian process (GP) priors. In particular, the second approach discussed below is based on a flexible Bayesian statistical framework for the approximation and analysis of computer simulator output, discussed in, for example, [8, 9, 11].

Both of the more general SEMs for average node speed presented below use GP methodology. Hence, we first provide a brief overview of GPs in the context of Bayesian non-parametric modeling. Consider a statistical model that depends on a function $g(z)$, which takes values, say, in (a subset of) R and is supported by (a subset of) R^q, i.e., $z = (z_1, \ldots, z_q)$. (For instance, for a non-parametric regression model with q covariates, $g(\cdot)$ plays the role of the regression function.) Specifying a flexible parametric form for the function $g(\cdot)$ might not be an easy task, and thus one might seek a more general *non-parametric* approach to modeling that treats the entire function $g(\cdot)$ as the unknown parameter. Such an approach is applicable to a larger collection of data examples; in fact, the shape of $g(\cdot)$ will be driven by the data in each particular application of the non-parametric model. But then, under the

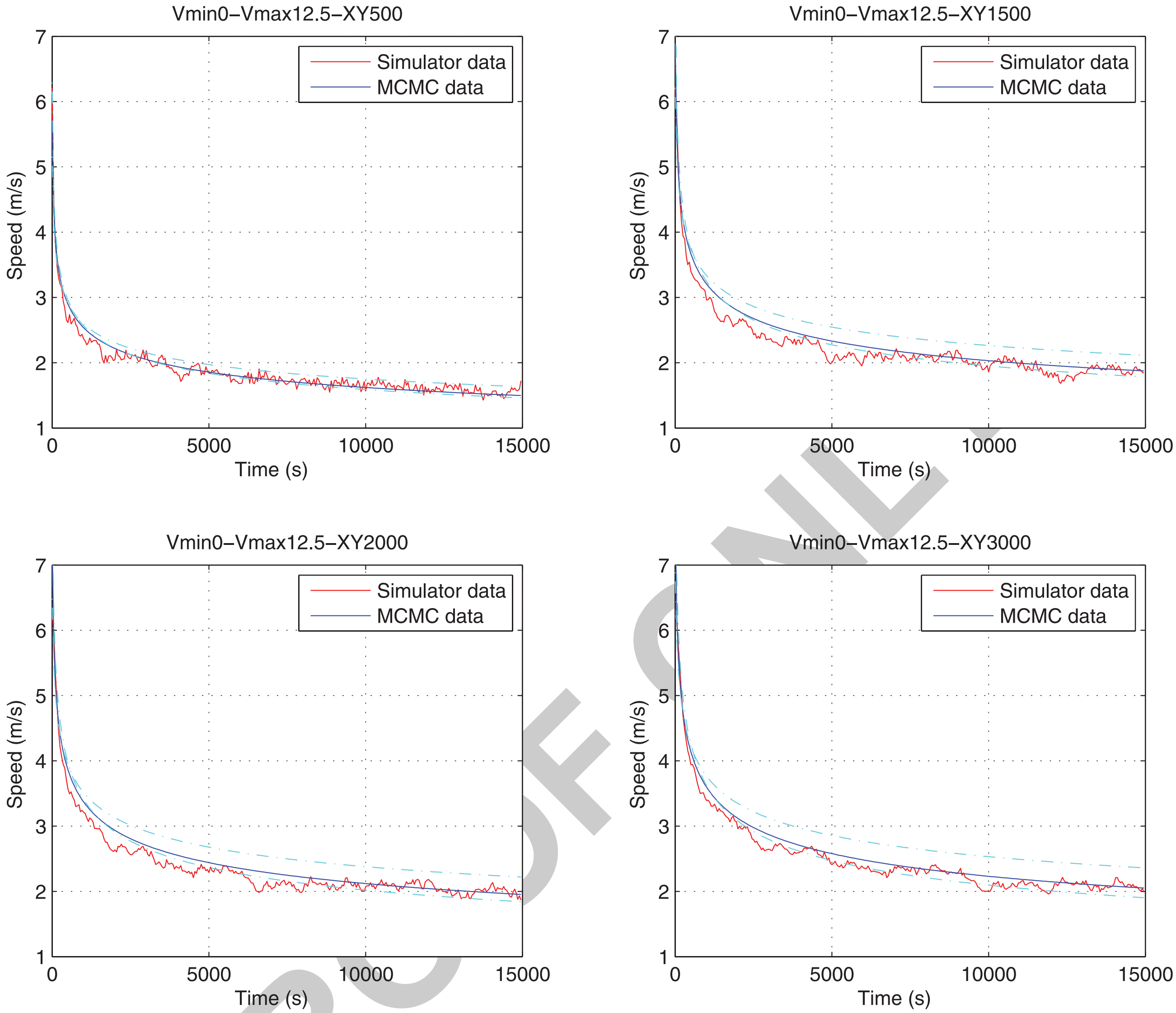


Figure 7. Inference: v_{max} 12.5 m/s

Bayesian framework, $g(\cdot)$ is a random function and a formal Bayesian approach requires a prior probability model for $g(\cdot)$. The area of Bayesian non-parametrics deals with the problem of placing priors on spaces of random functions; see, for example, [33] and [34] for reviews.

GPs offer an attractive approach to defining a prior for functions $g(\cdot)$, which are, in some sense, *smooth*, where the level of smoothness can be controlled by parameters of the GP prior. The GP is a stochastic process defined through a mean function, $\mu(z) = \mathrm{E}(g(z))$, and a covariance function, $C(z, z') = \mathrm{Cov}(g(z), g(z'))$. Used as a prior for $g(\cdot)$, for any choice of input points $(z_1, \ldots, z_n)$, the GP induces an n-variate normal prior distribution for $(g(z_1), \ldots, g(z_n))$ with mean vector $(\mu(z_1), \ldots, \mu(z_n))$, and covariance matrix with (i, j)-th element $C(z_i, z_j)$. The covariance function provides the means by which we can account for deviations from the prior mean function; it also determines how closely related realized values of $g(z)$ are for nearby values of z. It is typically taken to be isotropic, whence the variance $\mathrm{Var}(g(z))$ is constant for all z, and the correlation function, $\mathrm{Corr}(g(z), g(z'))$, depends on z and z' only through their distance. A standard choice for the correlation function is given by the product power exponential form, $\mathrm{Corr}(g(z), g(z')) = \exp\{-\sum_{j=1}^{q} \psi_j |z_j - z'_j|^{\alpha_j}\}$, where, for each input dimension, $\psi_j > 0$ controls the large scale variability (range of dependence), and $\alpha_j \in (0, 2$ controls the fine scale variability (smoothness) of $g(\cdot)$. Posterior inference un-

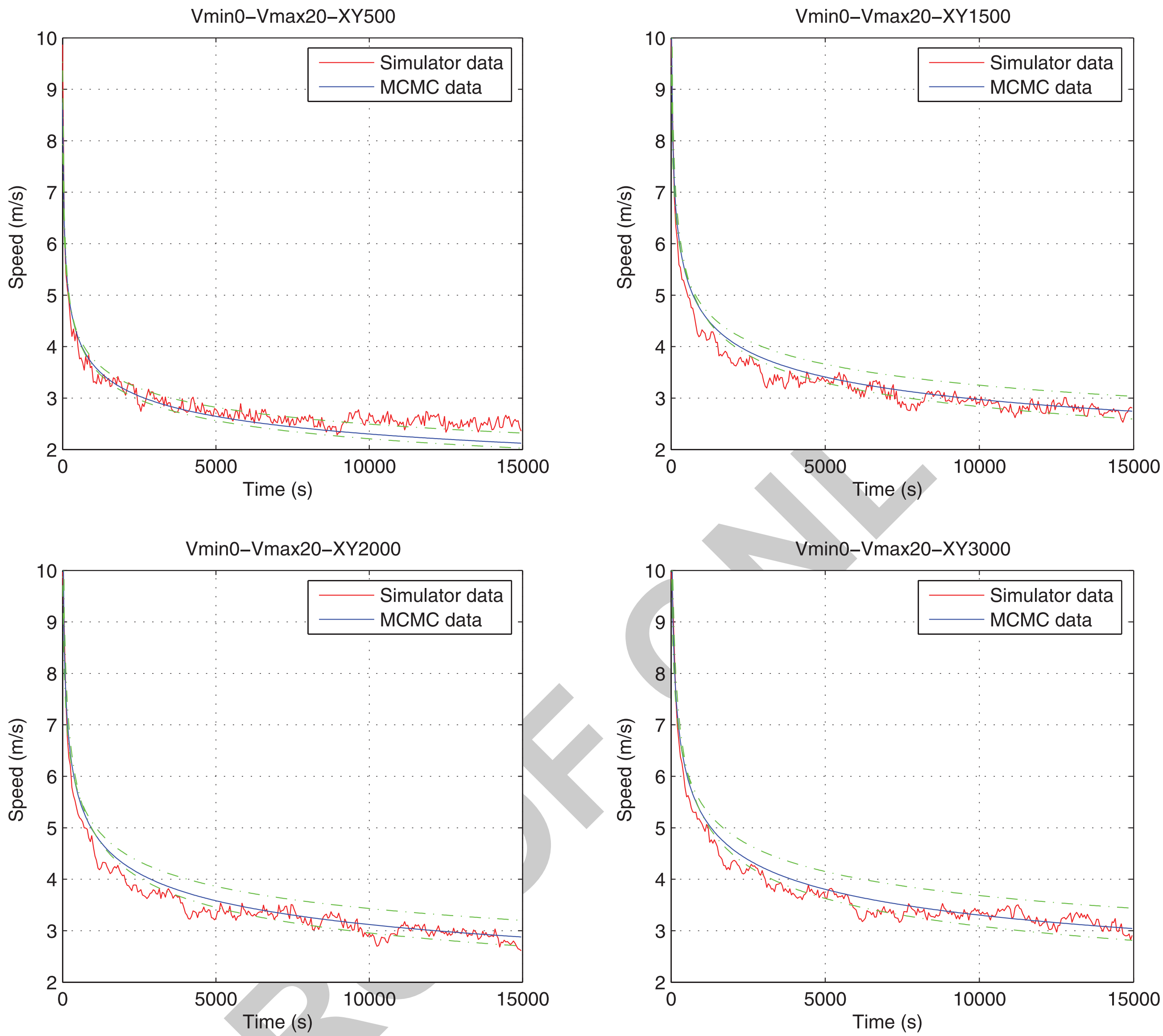


Figure 8. Inference: v_{max} 20 m/s

der GP priors is well-established, see, for example, [35] in the context of non-parametric regression, and [36] in the context of modeling for spatial and spatio-temporal data.

Turning to extensions of the SEM developed in Section 4, a first possibility involves retaining the non-linear regression form in equation (2), but now with independent GP priors for the functions $a(v, f)$, $b(v, f)$, $c(v, f)$. Each of these functions takes values in R^+ (and thus could be modeled by a GP prior on a logarithmic scale) and is supported by the subset of R^2 that defines the input space, i.e., the set of pairs (v, f) corresponding to plausible values for v_{max} and field size. Hence, this version replaces the parametric formulation in equation (3) for $a(v, f)$, $b(v, f)$ and $c(v, f)$, which is based on the 14-dimensional parameter vector $(\mathbf{a}, \mathbf{b}, \mathbf{c})$, with the nonparametric GP priors. The approach could be viewed as *semi-parametric*, since it utilizes the parametric regression form in equation (2) albeit with non-parametric priors for its three unknown functions. Under this semiparametric setting, the vector of unknown parameters comprises the collection, $\{(a(v_i, f_j), b(v_i, f_j), c(v_i, f_j)) : i = 1, ..., 10; j = 1, ..., 6\}$, of function values at all the combinations (v_i, f_j) for the inputs. Given the GP prior for the function $a(v, f)$, the induced prior for the vector of parameters $\{a(v_i, f_j) : i = 1, ..., 10; j = 1, ..., 6\}$ is multivariate normal. Similarly for the parameters $\{b(v_i, f_j) :$

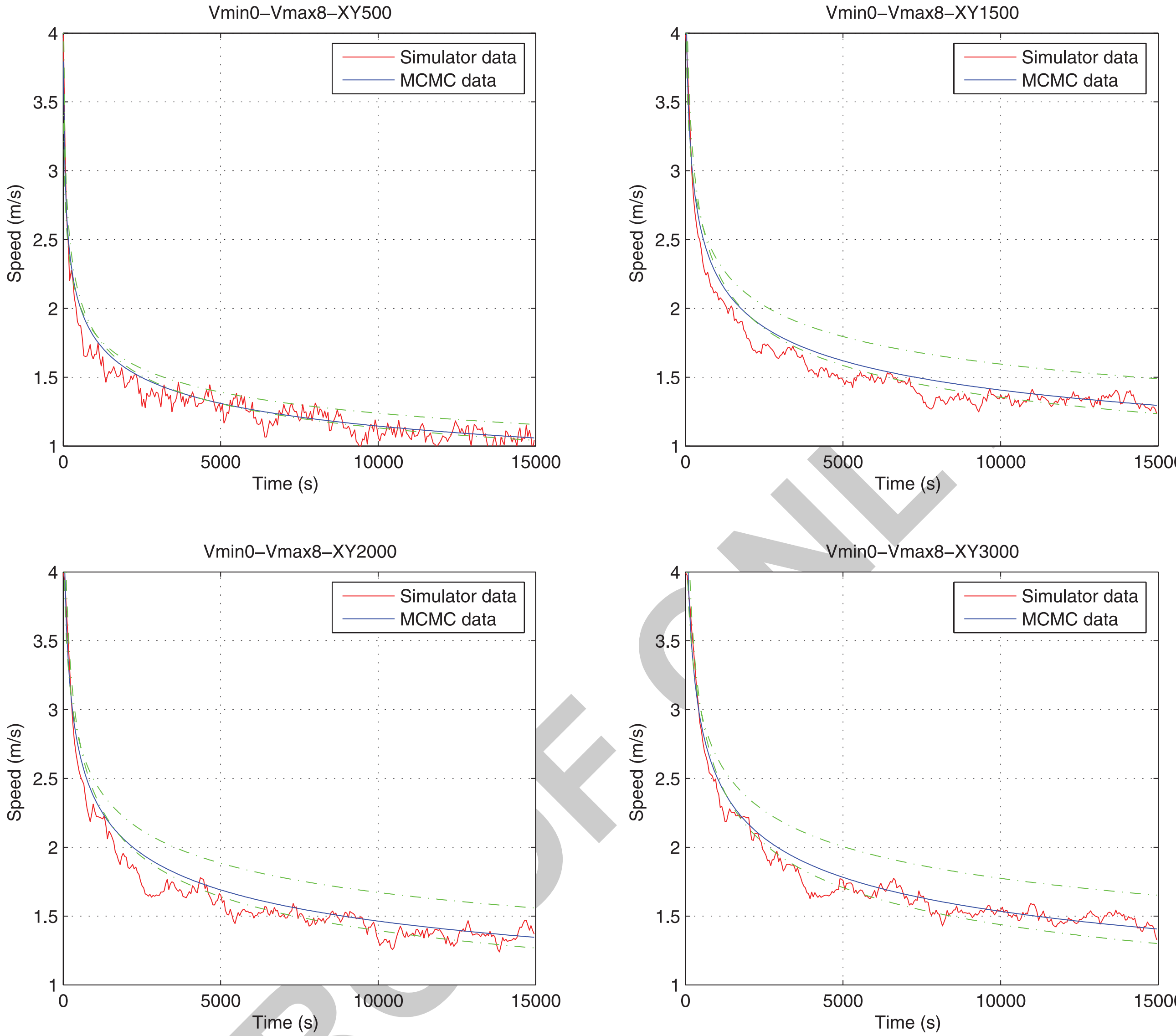


Figure 9. Validation v_{max} 8 m/s

$i = 1, ..., 10; j = 1, ..., 6\}$ and $\{c(v_i, f_j) : i = 1, ..., 10; j = 1, ..., 6\}$. The likelihood in equation (4) can be extended to include all these parameters. Again, an MCMC algorithm can be constructed to sample from the corresponding posterior. Moreover, the posterior of $a(v', f')$, $b(v', f')$, and $c(v', f')$ can be obtained for any pair of input values (v', f') that are not included in the 60 combinations (v_i, f_j) where the computer simulator was run. This interpolation step relies on the GP definition (specifically, the fact that the joint distribution of $\{a(v, f)\}$ or $\{b(v, f)\}$ or $\{c(v, f)\}$ is multivariate normal for any finite set of (v, f)) and can be performed using the posterior samples from the MCMC algorithm. Finally, by interpolating the functions $a(v, f)$, $b(v, f)$, and $c(v, f)$ over the input space, we can interpolate the function $g(t, v, f) = c(v, f)/(\{1+b(v, f)(t/1000)\}^{a(v,f)})$ over the input space and for the set of time points of interest; i.e., we can obtain the SEM approximation to average node speed for any combination of v_{max} and field size values and for any time point.

Note that both the parametric approach of Section 4 and the semiparametric approach discussed above to building an SEM for average node speed rely on the parametric form in equation (2), which was obtained after experimenting with several combinations of values for v_{max} and field size. Although it was demonstrated that this form yields a flexible SEM, the approach is specific to studying average node speed under the random waypoint

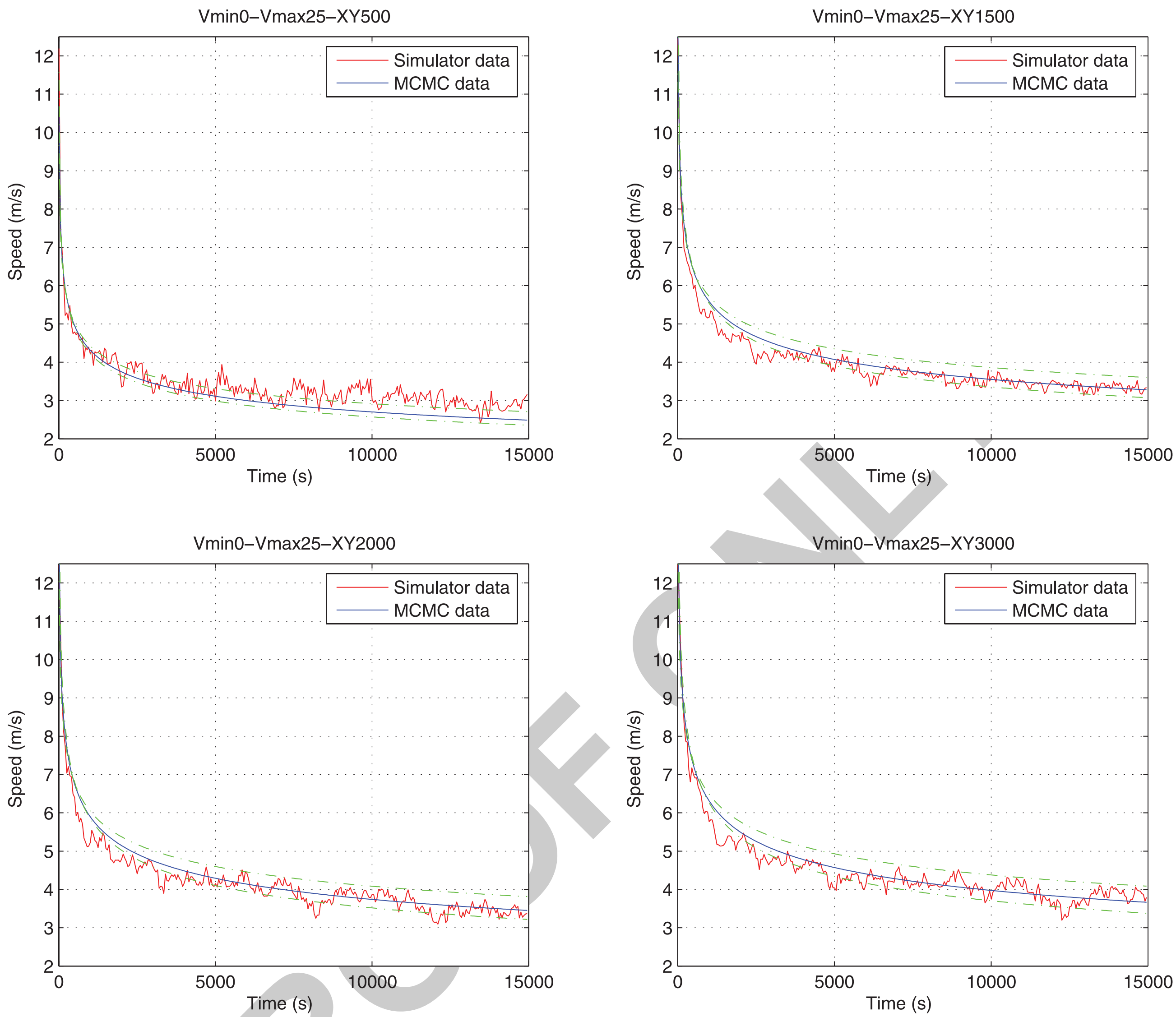
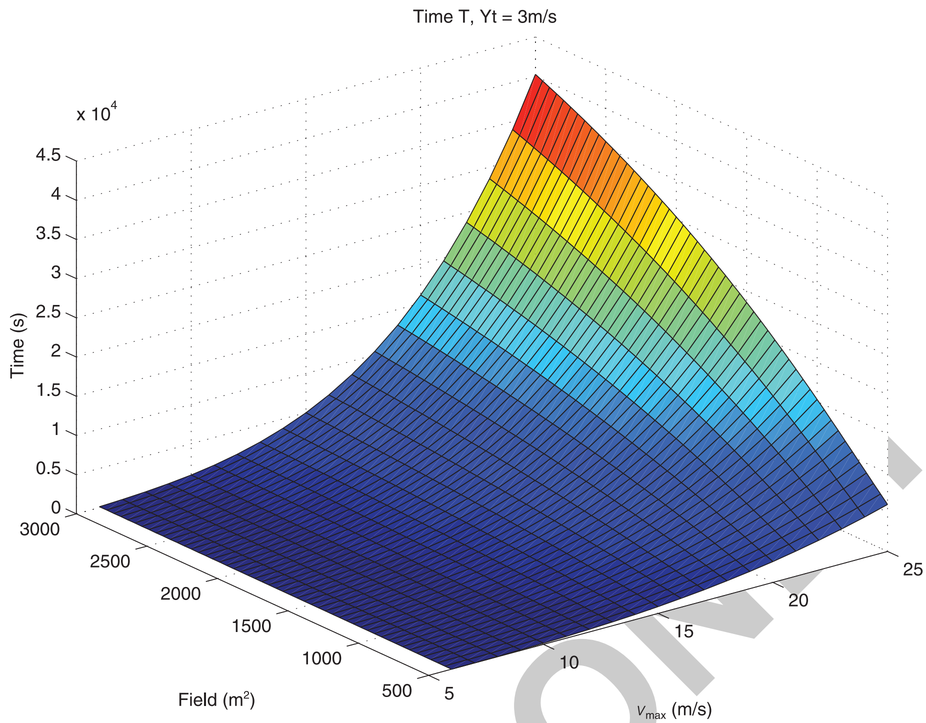
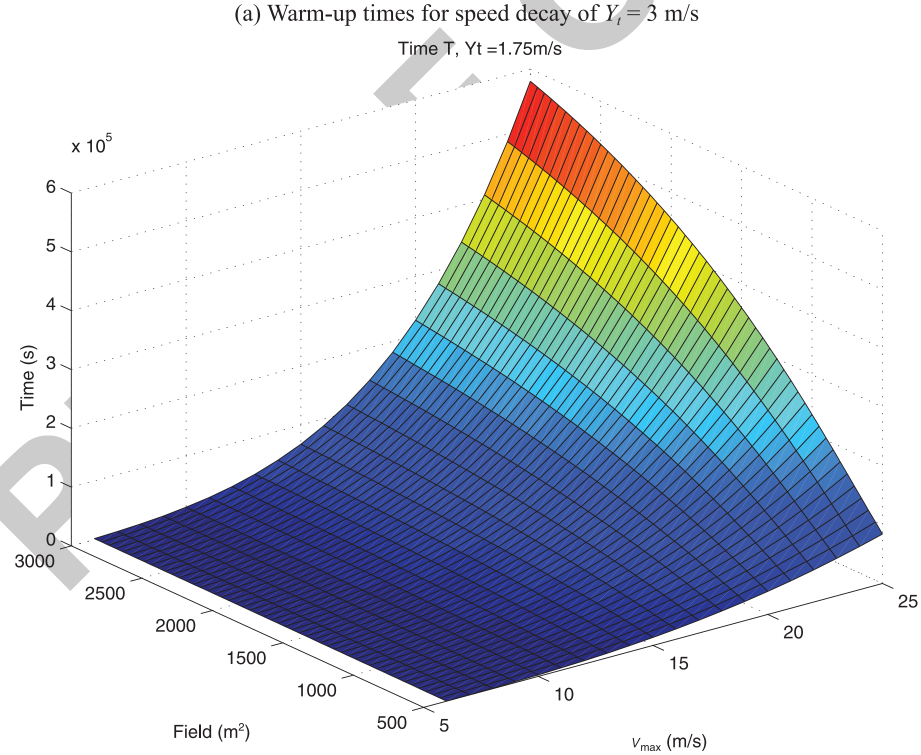


Figure 10. Validation v_{max} 25 m/s

mobility model. New formulations would be needed to handle different types of output from the random waypoint simulator, or, perhaps, for average node speed produced from a different mobility simulator. A more generic (fully nonparametric) approach emerges by applying the GP methodology directly to the function that describes the computer simulator output of interest. Illustrating again with average node speed under random waypoint mobility, consider first the output for a fixed time point t. Then a GP prior can be used for the function $g(v, f) \equiv g(t, v, f)$ that describes the expectation of average node speed for the specified time point t, again, under a normal error distribution. The SEM approximation in this case is obtained through the posterior of $g(v, f)$ for a grid of input points (v, f). This posterior can be sampled using MCMC techniques and the simulator data $y(v_i, f_j) \equiv y_t(v_i, f_j)$. Note that here the interpolation over the input space is directly for the average node speed output function. Of course, average node speed is functional output, i.e., for each pair (v_i, f_j) of v_{max} and field size values, the output is a curve over time. The most direct possibility to incorporate this functional output to the generic GP framework is to include time t as an input along with (v, f). Hence the input space is enlarged, becoming now a subset of R^3. Moreover, the number of observations from the simulator is now $60T$, where T is the number of time points chosen to represent the average node speed curves. Computational complexity grows rapidly with increasing T, since



(a) Warm-up times for speed decay of $Y_t = 3$ m/s



(b) Warm-up times for speed decay of $Y_t = 1.75$ m/s

Figure 11. Point estimates of $t_{warm-up}$ as a function of v_{max} and field size

implementation of posterior inference requires repeated inversion of matrices of dimension $60T$. Therefore, for this approach to be feasible, the points at which the function is recorded must be chosen carefully and their number must be fairly small. This can be accomplished in our case study, since, say, 10–20 time points are sufficient to capture the general shape of the average node speed curves; also, the number of (v_i, f_j) combinations can be reduced from 60 by choosing the v_i and f_j values based on one of the existing approaches to the design of computer experiments (e.g., [3–5]). This approach is discussed in [9, 14] (including computational simplifications to handle the increased number of input realizations) in the more general setting where field data is also available and it is of interest to use them for *validation* of the computer model simulator.

7. Conclusions

This paper conducts a case study of Statistical Equivalent Modeling applied to the Random Waypoint Mobility model used in network simulators. This novel modeling technique of characterizing the behavior of the Random Waypoint Mobility regime captures speed decay over time using maximum speed and terrain size as input parameters. A Bayesian approach to model fitting is employed to capture the uncertainty due to unknown parameters of the statistical model. The resulting posterior predictive distributions of the quantities of interest (i.e., average node speed) can be used to formally address the fit of the statistical model. We present results obtained from the model and evaluate its accuracy by validating it against data obtained from the simulator.

One of the main contributions of our random waypoint mobility SEM is that it offers an efficient alternative to circumventing recently uncovered anomalies of random waypoint mobility, one of the most widely used mobility models for evaluating the performance of multi-hop wireless ad hoc networks (MANETs). These anomalies include the fact that average node speed tends to zero as $t \to \infty$ and that node speed varies considerably from the expected average value for the time scales under consideration for most simulation analysis. Since our model characterizes average node speed as a function of time, it provides an accurate estimate of the time it takes for random waypoint mobility simulations to "warm up", i.e., reach steady state. Using this information, MANET protocol designers can continue to use the original random waypoint mobility model, discard simulation data produced during the "warm-up" period and still obtain accurate performance results for the protocols under study. This is an important contribution given that Random Waypoint Mobility is still, by far, the most widely used mobility model in the evaluation of MANETs [37–39].

We also show that our random waypoint mobility SEM is able to provide information on the warm-up period for different combinations of input parameters significantly faster than running pre-simulations of the mobility model for different input combinations. For instance, using our model, it took us 20 min to compute the point estimates of the warm-up period as a function of v_{max} and field size for two different values of speed decay. Using the same (reasonably fast) machine, it would take close to 200 times longer to run pre-simulations for the same number of combinations of v_{max} and field size values.

8. References

[1] Yoon, J., M. Liu, and B. Noble.2003. Random waypoint considered harmful. In *Proceedings of IEEE/INFOCOM*.

[2] Santner, T. J., B. J. Williams, and W. I. Notz. 2003. *The Design and Analysis of Computer Experiment*. Springer-Verlag, New York.

[3] Sacks, J., W. J. Welch, T. J. Mitchell, and H. P. Wynn. 1989. Design and analysis of computer experiments. *Statistical Science* 4, 409–35.

[4] Currin, C., T. Mitchell, M. Morris, and D. Ylvisaker. 1991. Bayesian prediction of deterministic functions, with applications to the design and analysis of computer experiments. *Journal of the American Statistical Association* 86, 953–63.

[5] Morris, M. D., T. J. Mitchell, and D. Ylvisaker. 1993. Bayesian design and analysis of computer experiments: Use of derivatives in surface prediction. *Technometrics* 35, 243–55.

[6] O'Hagan, A., M. C. Kennedy, and J. Oakley. 1998. *Uncertainty Analysis and other Inference Tools for Complex Computer Codes*. Oxford University Press.

[7] Craig, P. S., M. Goldstein, J. C. Rougier, and A. H. Seheult. 2001. Bayesian forecasting for complex systems using computer simulators. *Journal of the American Statistical Association* 96, 717–29.

[8] Kennedy, M. and A. O'Hagan. 2001. Bayesian calibration of computer models. *Journal of the Royal Statistical Society B* 63, 425–64.

[9] Bayarri, M. J., J. O. Berger, D. Higdon, M. C. Kennedy, A. Kottas, R. Paulo, J. Sacks, J. A. Cafeo, J. Cavendish, C. H. Lin, and J. Tu. 2002. A framework for validation of computer models. In *Proceedings of the Workshop on Foundations for V&V in the 21st Century*.

[10] Oakley, J., and A. O'Hagan. 2004. Probabilistic sensitivity analysis of complex models: A Bayesian approach. *Journal of the Royal Statistical Society, Series B* 66 751–69.

[11] Higdon, D., M. C. Kennedy, J. Cavendish, J. Cafeo, and R. Ryne. 2004. Combining field data and computer simulations for calibration and prediction. *SIAM Journal on Scientific Computing* 26, 448–66.

[12] Lee, H., B. Sansó, W. Zhou, and D. Higdon. 2006. Inferring particle distribution in a proton accelerator experiment. *Bayesian Analysis* 1, 249–64.

[13] Bayarri, M. J., J. O. Berger, G. Garcia-Donato, J. Palomo, J. Sacks, D. Walsh, J. Cafeo, and R. Parthasarathy. 2005. *Computer Model Validation with Functional Output*, Technical Report. National Institute of Statistical Sciences.

[14] Bayarri, M., J. Berger, M. Kennedy, A. Kottas, R. Paulo, J. Sacks, J. Cafeo, C. Lin, and J. Tu, 2005. *Bayesian Validation of Computer Models for Smooth Functional Data in Hierarchical Contexts*, Technical Report AMS2005-11. Department of Applied Mathematics and Statistics, University of California, Santa Cruz.

[15] NS-2, VINT Project, USC/ISI, available from http://www.isi.edu/nsnam/ns.

[16] Zeng, X., R. Bagrodia, and M. Gerla. 1998. GloMoSim: a library for parallel simulation of large-scale wireless networks. In *Proceedings of PADS'98*, software available from http://pcl.cs.ucla.edu/projects/domains/glomosim.html.

[17] Qualnet Wireless Simulator, Scalable Network Technologies, available from http://www.qualnet.com.
[18] Opnet, OPNET Technologies, available from http://www.opnet.com/.
[19] Johnson, D., and D. Maltz. 1996. Dynamic source routing in ad hoc wireless networks. In *Mobile Computing*, Chapter 5. Kluwer Academic Publishers.
[20] Camp, T., J. Boleng, and V. Davies. 2002. A survey of mobility models for ad hoc network research. *Wireless Communication and Mobile Computing (WCMC): Special Issue on Mobile Ad Hoc Networking* 2(5), 483–502.
[21] Davies, V. 2000. *Evaluating Mobility Models within an Ad Hoc Network*. Masters Thesis, Colorado School of Mines.
[22] Hong, X., M. Gerla, G. Pei, and C. Chiang. 1999. A group mobility model for ad hoc wireless networks. In *Proceedings of ACM/IEEE MSWiM'99*, Seattle, WA, pp. 53–60.
[23] Bettstetter, C. 2001. Smooth is better than sharp: A random mobility model for simulation of wireless networks. In *Proceedings of ACM MSWiM 2001*.
[24] Lee, S., W. Su, J. Hsu, M. Gerla, and R. Bagrodia. 2000. A performance comparison study of ad hoc wireless multicast protocols. In *Proceedings of the IEEE Infocom 2000*.
[25] Perkins, D. D., H. D. Hughes, and C. B. Owen. 2002. Factors affecting the performance of ad hoc networks. In *Proceedings of the IEEE International Conference on Communications*.
[26] Perkins, C., and E. Royer. 1999. Ad-hoc on demand distance vector routing. In *Proceedings of the IEEE Workshop on Mobile Computing Systems and Applications (WMCSA)*, pp. 90–100.
[27] Bettstetter C., and C. Wagner. 2002. The spatial node distribution of the random waypoint mobility model. In *Proceedings of German Workshop on Mobile Ad Hoc Networks (WMAN)*, Ulm, Germany.
[28] Santi, P., and G. Resta. 2002. An analysis of the node spatial distribution of the random waypoint mobility model of ad hoc networks. In *Proceeding of ACM Workshop on Principles of Mobile Computing(POMC)*, Toulouse, France.
[29] Navidi, W., and T. Camp 2003. Stationary distributions for the random waypoint mobility model. *IEEE Transactions on Mobile Computing*.
[30] Yoon, J., M. Liu, and B. Noble. 2003. Sound mobility models. In *Proceedings of ACM International Conference of Mobile Computing and Networks (MOBICOM)*.
[31] Boudec, J. L. and M. Vojnovic. 2005. Perfect simulation and stationarity of a class of mobility models. In *Proceedings of the IEEE Infocom 2005*.
[32] Gamerman, D. 1997. *Markov Chain Monte Carlo*. Chapman and Hall, London, UK.
[33] Walker, S. G., P. Damien, P. W. Laud, and A. F. M. Smith. 1999. Bayesian nonparametric inference for random distributions and related functions (with discussion). *Journal of the Royal Statistical Society, Series B* 61, 485–527.
[34] Müller, P., and F. Quintana. 2004. Nonparametric Bayesian data analysis. *Statistical Science* 19, 95–110.
[35] Neal, R. 1998. Regression and classification using Gaussian process priors. In *Bayesian Statistics 6: Proceedings of the Sixth Valencia International Meeting*, pp. 475–501.
[36] Banerjee, S., B. P. Carlin, and A. E. Gelfand. 2004. *Hierarchical Modeling and Analysis for Spatial Data*. Chapman and Hall.
[37] Bai, N. S. F., and A. Helmy. 2003. Important: A framework to systematically analyze the impact of mobility on performance of routing protocols for adhoc networks. In *Proceedings of IEEE INFOCOM*.
[38] Barrett, C., M. Drozda, A. Marathe, and M. Marathe. 2003. Characterizing the interaction between routing and mac protocolsin adhoc networks. In *Proceedings of the ACM International Symposium on Mobile Ad Hoc Networking and Computing*, Lausanne, Switzerland, pp. 92–103.
[39] Vaishampayan R., and J. Garcia-Luna-Aceves. 2004. Efficient and robust multicast routing in mobile ad hoc networks. In *Proceedings of 1st IEEE International Conference on Mobile Ad-hoc and Sensor Systems (MASS)*.

Kumar Viswanath *received his MS degree in Electrical Engineering from the University of Florida, Gainesville and his PhD degree in Computer Engineering at the University of California, Santa Cruz. He is currently employed as a research scientist in the Security Technologies group at NTT Multimedia Communication Labs. He has been active in the field of internet working and group communications in ad-hoc networks since 1999. His research interests include network protocol design, distributed systems, security technologies and wireless systems in general.*

Athanasios Kottas *is currently Assistant Professor in the Department of Applied Mathematics and Statistics, University of California, Santa Cruz. He obtained his PhD in Statistics in 2000, and, from 2000 to 2002, he worked as Visiting Assistant Professor in the Institute of Statistics and Decision Sciences, Duke University. His research interests include Bayesian nonparametric modeling for computer model experiments, regression problems, survival analysis, and spatial statistics, with applications in ecology and engineering.*

Katia Obraczka *is currently an Associate Professor of Computer Engineering at the University of California, Santa Cruz. Before joining UCSC, she held a research scientist position at USC's Information Sciences Institute and a research faculty appointment at USC's Computer Science Department. Her research interests include computer networks, more specifically, network protocol design and evaluation for wireline-as well as wireless (in particular, multi-hop ad hoc and sensor) networks, distributed systems, and Internet information systems.*

Bruno Sansó *is currently an Associate Professor in the Department of Applied Mathematics and Statistics of the University of California Santa Cruz. He obtained his PhD in 1992 and, until 2001 he was a Professor at Universidad Simon Bolivar, Caracas, Venezuela. His research interests include Bayesian statistical methods, statistical models for space-time data, inference for the output of computer simulators and inference for climate projections.*