Modeling Spatial Node Density in Waypoint Mobility

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***Abstract*—This paper introduces a modeling framework to analyze spatial node density in mobile networks under "waypoint"-like mobility regimes. The proposed framework is based on a set of first order ordinary differential equations (ODEs) that take as parameters (1) the probability of going from one subregion of the mobility domain to another and (2) the rate at which a node decides to leave a given subregion.**

We validate our model by using it to describe the steady-state behavior of real user mobility recorded by GPS traces in different scenarios. To the best of our knowledge, this is the first node density modeling framework generic enough that can be applied to any "waypoint"-based mobility regime.

I. Introduction

One way to characterize and describe mobility is through the *spatial density* of mobile nodes. Spatial node density can be defined as the number of nodes located in a given unit area and has significant impact on fundamental network properties, such as connectivity and capacity, as well as on core network functions, e.g., medium access and routing. To date, only a few efforts have focused on modeling spatial density. Notable examples include [3, 4, 5]. However, most previous work have been focusing exclusively on the Random Waypoint (RWP) model [1].

In this paper, we introduce an Ordinary Differential Equation (ODE) framework to mathematically model spatial node density under different mobility regimes that are in general "waypoint"-based. More specifically, our model describes node density's steady-state behavior under mobility which is characterized by having nodes probabilistically choose the next destination, or waypoint, based on some probability density function. We contend that waypoint mobility is one way to describe forms of human mobility. Therefore, we validate our model by using it to describe the steady-state behavior of real user mobility recorded by GPS traces in different scenarios and compare the results against the corresponding traces. Moreover, we present comparative results for steady-state spatial distribution analysis of a number of synthetic waypoint mobility regimes. To the best of our knowledge, this is the first node density modeling framework generic enough that it can be applied to any waypoint-based mobility regime. As an example, we use our framework to model the well-known RWP mobility regime. Our model confirms the well-known result showing that node density's steady-state behavior under RWP mobility tends to homogeneity, as defined in [6][2]. Furthermore, we also use our framework to model mobility regimes based on the *preferential attachment* principle [2]. We show through the application of our proposed model that using preferential attachment to model human mobility leads to undesirable steady-state behavior. More specifically, our model shows that, at steady state, the original spatial node density distribution is not preserved and exhibits behavior similar to random mobility a la Random Waypoint regime. This behavior has been observed empirically in [7], where we show that, instead, real human mobility exhibits "persistent" density heterogeneity.

We introduce the first generic spatial node density modeling framework for waypoint mobility regimes and validate our framework by applying it to real user mobility traces.

The remainder of this paper is organized as follows. Our ODE model is presented in detail in Section II, how its parameters are set, and our implementation. Section III show the validation of our proposed framework towards modeling real human mobility.Finally, Section V concludes the paper with a discussion of future work.

II. Proposed Model and Framework

Our objective is to model the spatial node density of a mobile network. We assume a waypoint-based mobility pattern, where nodes stay in a given location i for a given period of time and choose to leave i towards another location j with probability p_{ij}. Once the node arrives at j, the process restarts.

A. ODE Framework

Assume a mobile network composed of m mobile nodes, where all nodes are capable of moving around inside a

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[2]The use of the term "homogeneous node distribution" refers here to the fact that there is no significant concentration of nodes (clusters), and should not be mistaken with uniform distribution normally used to model the choice of next destination, speed and pause time in random mobility models.

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delimited area a. Now assume this area is divided into equally sized square subareas of size $l \times l$, defined here as *cells*. The mobile nodes can then choose to move from cell to cell with a given probability. Let $X(t)$ be the stochastic process that determines which cell a mobile node chooses at time t. We can write then $p_{ij} = P\{X(t) = i \mid X(t+\gamma) = j\}$, as the *transition probability*, which is the probability that a node in cell i, at time t, is going to choose to go to cell j at time $(t+\gamma)$, after some time step γ.

Thus, we are interested in the average number of nodes in each cell i, represented by the component $N_i(t)$ $\forall i \in \{1, ..., n\}$ of the state vector $N(t) \in \mathbb{R}^{n\times 1}$, where n is the total number of cells for the desired scenario.

The variation in the number of nodes at each cell $\dot{N}_i(t) = \frac{dN_i(t)}{dt}$ is simply the difference between nodes arriving in cell i and the ones departing from the same cell at time t, as expressed in Equation 1.

$$\dot{N}_i(t) = \underbrace{\sum_j p_i \mu_j N_j(t)}_{\text{Arriving at cell i}} - \underbrace{\sum_j p_j \mu_i N_i(t)}_{\text{Departing from cell i}} , \qquad (1)$$

where, μ_i is the rate at which nodes decide to leave cell i towards another cell, which allow us to write $\mu_{ij} = p_{ij}\mu_i$ as the rate at which nodes in cell i decide to leave this cell towards cell j. We can also define the arrival rate in cell i as the sum of the departing rates of all nodes going from cell j to cell i, over all possible values of j, including $j = i$, since we allow transitions from a cell to another position in itself.

In reality we observe that nodes prefer some cells over others and some transitions over others. The probability of choosing a destination and the rate at which nodes depart from that destination depends on how popular that destination is and what are the nodes' interests in each destination. For example, nodes moving around on a campus environment may go very often from the cafeteria to the classroom, but not so often from the cafeteria to the library. This means that $p_{\text{cafeteria,classroom}} > p_{\text{cafeteria,library}}$. Moreover, since people might tend to stay inside the library for longer than in the cafeteria, the relationship between the departure rate from this two locations might be such as $\mu_{\text{cafeteria}} > \mu_{\text{library}}$.

In order to simplify our model, more specifically the choice of the parameters (departure rates and transition probabilities), we define the rate μ_i as the inverse of the average time spent by the nodes in cell i. We also considered the transition probabilities independent of where the transition originated. This means that the probability of going from cell j to cell i is the same probability of simply choosing cell i as the next destination for all j. We then make $p_{ji} = P\{X(t) = j \mid X(t+\gamma) = i\} = P\{X(t+\gamma) = i\} = p_i$.

Moreover, in order to validate our model we have chosen to extract the model parameters from— and compare our results with— real live GPS traces, where the number of nodes in the system remains constant during the whole duration of the trace. For that reason, in the results we present in Section III-C we used a slightly simplified version of our model, where $\lambda_0 = \mu_0 = 0$. Equation 1 gives this version of our ODE model.

B. Implementation

In this section we present a vectorized version of Equation 1, so that we could implement it on MATLAB [9]. We used a 4th order Runge-Kutta ODE solver, native to the platform, to do so.

We start by defining a matrix $A \in \mathbb{R}^{n\times n}$ as a parameter matrix given by $A = P \times M$. $P \in \mathbb{R}^{n\times 1}$ is a column vector containing in every ith position the probability p_i of a node choosing cell i as the next destination, and $M \in \mathbb{R}^{1\times n}$ a row vector containing in every ith position the rate μ_i at which nodes choose to leave cell i. The components of matrix A, resulting from this multiplication are $a_{ij} = p_i\mu_j$.

Thus, it is possible to write Equation 1 for $\dot{N}(t) \in \mathbb{R}^{n\times 1}$ in its equivalent vectorized form as follows:

$$\dot{N}(t) = \underbrace{A \times N(t)}_{\text{Arriving}} - \underbrace{\Big((A^T \times \vec{\mathbb{1}}) \cdot N(t)\Big)}_{\text{Departing}}, \qquad (2)$$

where A^T is the transpose of matrix A, that we multiply by $\vec{\mathbb{1}} \in \mathbb{R}^{n\times 1}$, a column vector of ones, to give us a resulting $n \times 1$ column vector in which every component i represents the summation of all the components of the ith row of matrix A^T. After that, we perform a component wise multiplication with the state vector $N(t)$, which gives us the number of nodes departing from a given cell. That represents the second summation in the right-handed side of Equation 1.

III. Spatial Node Density of Human Mobility

We validate our model using real mobility traces; in other words, we show how the model can be applied to describe the steady-state behavior of spatial node density associated with human mobility. Three real GPS traces were used in our validation. These traces were collected in scenarios that are quite diverse, namely a city park, a university campus, and a state fair. We describe these traces in detail below as well as how we use information from the traces to estimate the parameters of our ODE framework.

A. Mobility Traces

Table I summarizes the GPS traces in terms of number of users, duration of the trace, and GPS sampling period.

Quinta, refers to the "Quinta da Boa Vista Park" trace, first presented in [10]. It is a GPS trace collected at a park in the city of Rio de Janeiro, Brazil. The park has many trees, lakes, caves, and trails. It houses the National Museum of Natural History and the city Zoo. The *KAIST* trace [11], on the other hand, is a GPS trace collected at the KAIST University campus in Daejeon, South Korea. The *Statefair* trace, also available at [11], is yet another mobility scenario showing daily GPS track logs collected from the NC State Fair held in North Carolina, USA.

Trace	# users	Duration	Samples
Quinta [10]	97	900s	1s
KAIST [11]	78	5000s	10s
Statefair [11]	19	8000s	10s

TABLE I
SUMMARY OF THE GPS TRACES STUDIED.

B. Parameter Estimation

We extract from the traces the distributions of *speed*, *pause time*, and *node density*. We use the trace's sampling period, e.g., for example, in the *Quinta* trace, the sampling period is $T = 1$ seconds. Node speed is defined as $\frac{d}{\Delta t}$ where d is the distance traveled between two consecutive entries in the GPS trace at times t_1 and t_2 and $\Delta t = t_2 - t_1$. Pause time is defined as $P = \Delta t$, if $d < \textit{threshold}$, or zero otherwise. The threshold is used to account for GPS error. We set this threshold to be 2 meters for KAIST and Statefair traces and 0.5 meter for the Quinta trace, due to jitter in GPS update frequency.

To extract spatial node density, the area covered in the trace is divided into squared cells of 140 x 140 meters. The choice of cell size was based on empirical observations, i.e., we picked a cell size that provided both adequate resolution as well as clustering. An alternate approach could be based on identifying "attraction zones", as was done in [12]. This is one of the topics of future work we plan to address. At the limit, i.e., where the cell is either infinitesimal (lower limit) or the size of the whole area (upper limit), all the traces and synthetic mobility regimes would have the same relative spatial density, namely one or zero nodes per cell for the lower limit and all the nodes in the same (unique) cell for the upper limit.

After dividing the area into cells, we took a snapshot of the number of nodes at every cell every T seconds. The value of $T = 10$ was used since, for the size of the cells and the speeds sampled from the traces, a node could not on average change between more than two cells during T. For every cell, at every interval T we counted the number of nodes in each cell. We then averaged the number of nodes in each cell over the course of the whole duration of the trace. The result is what we refer to as *Intensity Map (IM)* which we use to estimate the probability a node will choose a given cell as its next destination.

In the case of real mobility, e.g., as described by GPS traces, we set the *probabilities of choosing a given cell*, p_i of our ODE model to be the normalized value of the IM for cell i, such that $p_i = \frac{IM(i)}{\sum_j IM(j)}$, where IM($i$) is the intensity in cell i.

The rate μ_i, as mentioned before, is computed as the inverse of the average time spent by the nodes in cell i. This time has two components. The time spent by the node moving towards or from a given point in the cell, and the time spent in pause at this point, which reflect both main basic parameters of human mobility, speed and pause time. This two components were empirically measured from the GPS traces and used to compute μ_i.

C. Results

As highlighted in previous sections, the goal of our model is to describe the steady-state behavior of spatial node density in *waypoint*-like mobility regimes in which: (1) a node chooses its next destination following some given probability distribution, (2) moves to that destination, (3) pauses for some time, and (4) repeats from step (1).

Spatial node density is defined as the percentage of cells containing $\geq k$ nodes. It can also be expressed as the probability of finding a cell containing $\geq k$ nodes. It describes the degree of "clustering" exhibited by mobility regimes and can be used to evaluate how close to reality a given synthetic mobility regime is as far as its ability to mimic the degree of clustering exhibited by real mobility.

We followed the guidelines presented in Section III-B to estimate the parameters of our model for each of the traces studied. Figures 1, 2 and 3 plot spatial node density in the Quinta, KAIST, and Statefair scenarios, respectively. Each figure shows three curves plotting the spatial density: (1) at the beginning of the trace, (2) at the end of the trace, i.e., at 900 seconds for the Quinta Trace, 500 seconds for KAIST, and 8000 seconds for the Statefair, and (3) by applying our ODE framework. Note that the plots for the $KAIST$ and $Statefair$ traces are zoomed into the region of interest. In those two plots, the only point not shown is $k = 0$, where the percentage of cells containing 0 or more nodes $P[k \geq 0]$ is the same for every curve and it is, of course, equal to 100%.

The *largest* deviation of our ODE model from the final density distribution measured from the traces, for any value of k at any instant is 5.36%, 0.58% and 7.52%; the average deviation from the initial distribution measured in all the instants for all values of k is 1.45%, 0.06% and 2.02% for the Quinta, KAIST and Statefair traces respectively.

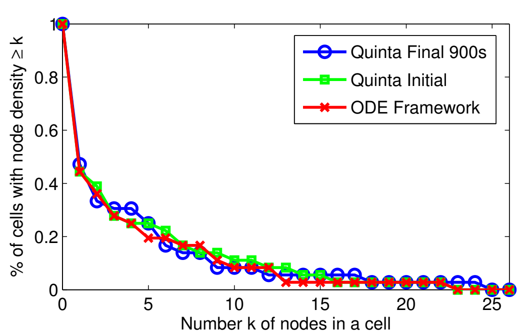

Fig. 1. Initial and final spatial node density distribution for the Quinta trace, and the respective steady-state density distribution using the proposed ODE framework.

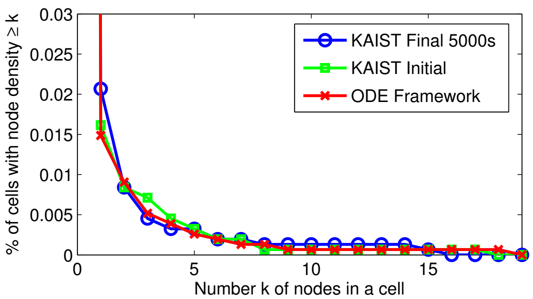

Fig. 2. Initial and final spatial node density distribution for the KAIST trace, and the respective steady-state density distribution using the proposed ODE framework.

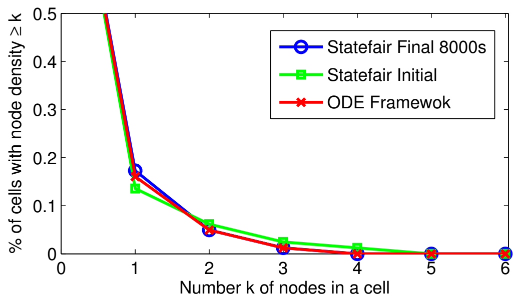

Fig. 3. Initial and final spatial node density distribution for the Quinta trace, and the respective steady-state density distribution using the proposed ODE framework.

IV. Related Work

Mobility models are vital in the design, testing, and evaluation of wireless networks and their protocols. Recently, network researchers and practitioners have been trying to use more realistic scenarios to drive the evaluation of wireless network protocols. This motivated initiatives such as the CRAWDAD [16] trace repository, which makes real traces available to the networking community. These traces can then be used to run trace-driven simulations. Even though initiatives like CRAWDAD have greatly increased availability of real traces, relying exclusively on traces to design and evaluate network protocols would not allow a broad enough exploration of the design space.

A number of efforts have proposed mobility models based on realistic mobility patterns [17]. Notable examples include [18, 19, 20, 21]. More recent work focuses on the "scale-free" properties observed in many real networks like the Internet, the Web, and some social networks, to name a few. The seminal work of Barabási and Albert [2] proposes a model that generates scale-free networks, i.e., networks whose node degrees follow a power law distribution, and demonstrate that many real large networks are scale free, that is, the node degree in the network graph follows a power law. The authors discuss the mechanism responsible for the emergence of scale-free networks and argue that understanding this problem will require a shift from modeling network topology to modeling "network assembly and evolution". To this end, they define the Barabási-Albert model based on *growth* and *preferential attachment*, in order to generate more realistic simulated network connectivity. Growth refers to the fact that the number of nodes in the network increases over time, where a new node is placed with m edges connecting it to other m nodes. Preferential attachment means that a node will choose to connect to another node i with probability $\Pi(k_i) = \frac{k_i}{\sum_j k_j}$ based on the degree k_i of node i and any node j connected to node i.

In other words, this key concept underpinning the Barabási-Albert model, referred to as the *preferential attachment* principle, states that "the more connected a node is, the more likely it is to receive new links". Several recently proposed mobility models (e.g., [22, 12, 8, 23, 24, 25]), try to mimic real human mobility by following the preferential attachment principle: they define *attraction points*, whose probabilities of attracting other nodes increase as more nodes congregate around them. The main goal of these preferential attachment based approaches is to try to maintain the non-homogeneity of the spatial node density observed in real live mobility traces, also maintaining the clusters of nodes naturally formed in real applications. Our model can be used to study the steady-state of such mobility regimes to show how the long run, they are very similar to the RWP model in terms of node density distribution [7].

In this work, the main focus is over spatial node density. Most previous work on modeling node spatial density have been focusing on the RWP model, rather then trying to characterize it in real applications. In [4], for example, analytical expressions are derived for the spatial density distribution that results from using the RWP model on simulations. One-dimensional case is analyzed and an approximation for the two-dimensional case is also given. They also analyze the concept of attraction areas in a modified version of the RWP regime. One other example of analytical work towards modeling steady-state behavior of the RWP can be found in [5]. In that work, authors derive stationary analytical expressions for node density and node velocity.

Bettstetter et al. point out that random mobility leads to homogeneous node distributions [6]. They propose a method that creates initial non-homogeneous node distributions and in [3], authors analyze the impact of random mobility in the inhomogeneity of spatial distributions via simulations.

The proposed first order ODE framework described in this paper differs from previous work on modeling spatial distribution, first in its nature, the fact that it uses ODEs as a tool for studying density distributions (never used before). Secondly, also by the fact that our approach is the first that is generic enough that can be applied in any mobility regime that bases its behavior on a waypoint-like movement. By waypoint-like movement we mean, "pause - choose the next destination following some given probability distribution - move to next destination in a straight line and constant speed - repeat". It is yet, capable of faithfully replicate the stead-state behavior of spatial distributions in real scenarios, as we show through our results, validating our model against real live traces.

V. Conclusions

We investigated in this paper the spatial density properties of waypoint-like mobility, which can describe some instances of human mobility. To this end, we developed an Ordinary Differential Equations (ODEs) framework to model spatial node density. With our model, it is possible to study the steady-state behavior of node density for waypoint-based mobility regimes as well as real mobility described by GPS traces. We validate our approach by comparing its results against GPS traces.

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